ELECTROSTATIC scaling in MEMS

Scaling Laws in Micro & Nanosystems

H. Shea

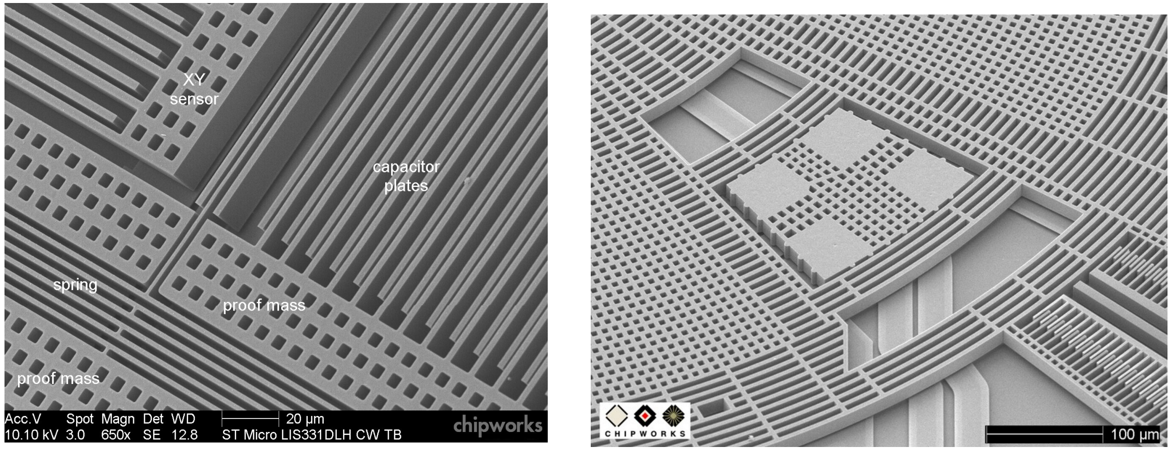
v. 2025

Scaling in Electrostatics

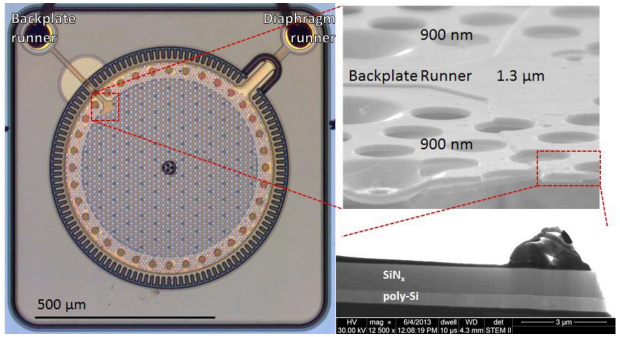
- Parallel plate Capacitor
- Energy density in capacitors, Paschen curve
- Parallel plate actuator, pull-in, spring softening
- Zipping actuators
- Comb drive
- Resonators

Concepts to master - Electrostatics

- Capacitive actuator
  - Energy density in a capacitor (parallel plate and comb)
  - Force derived from energy
  - Scaling of Force with geometry
  - Spring constant derived from force
    - Spring softening (parallel plate vs. comb drive)
    - Pull-in instability in parallel plate
    - Failure mode comb-drive
  - Paschen curve in air and implications for scaling
  - Resonators
    - Drive and sense principles
    - Temperature drift and solutions

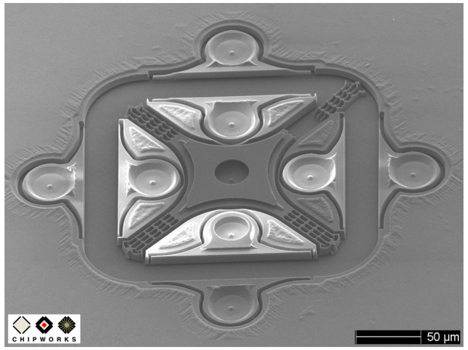


Inertial sensing: accelerometer and gyro (photos of ST micro devices)

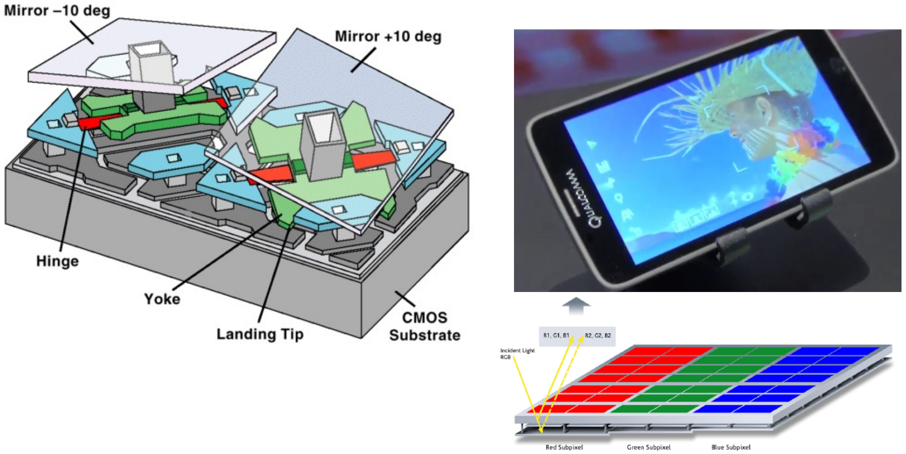


Microphone

M. Broas, et al , *2015 (ECTC)*

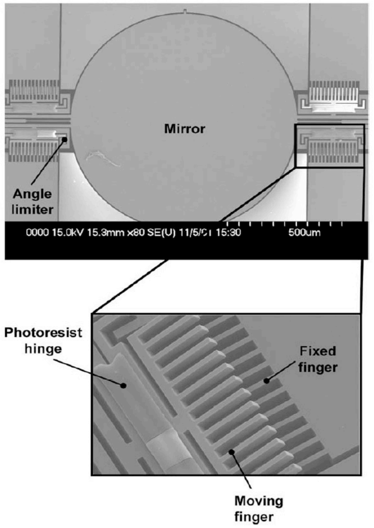


RF resonator (SiTIme)



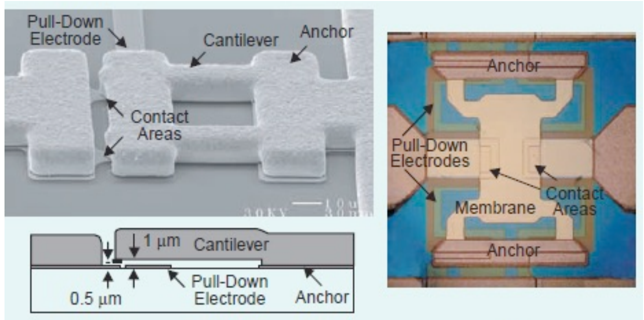
Displays (TI DMD, Qualcomm Mirasol)

Electrostatic MEMS are everywhere!



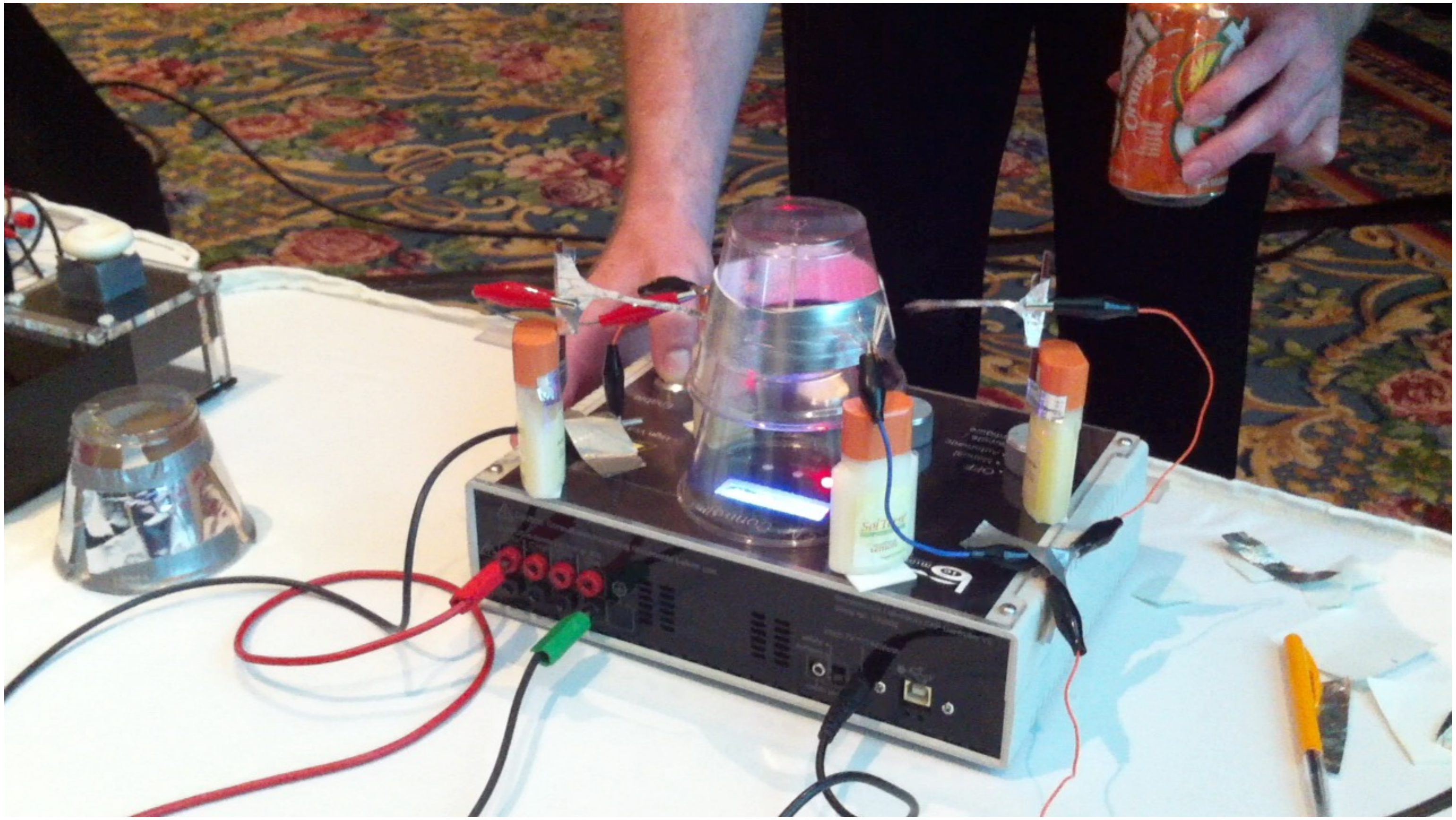
MEMS optical scanner

(Hah et al, 2004) IEEE Journal of Selected Topics in Quantum Electronics



RF switch (Analog devices, Raytheon)

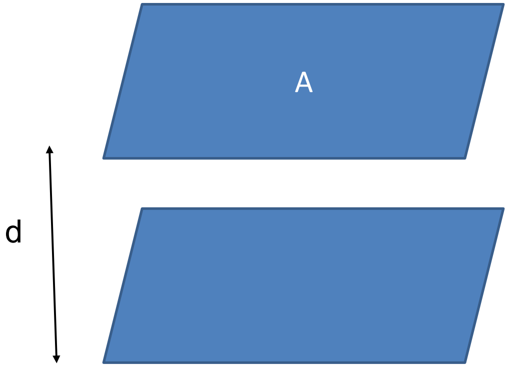
cm-scale electrostatic motor !
(made from parts stolen from a motel bathroom, plus a HV supply)



Biomimetic lab, U. Auckland, NZ
O'Brien et al., . "Rotating turkeys and self-commutating artificial muscle motors."
Applied Physics Letters 100, no. 7 (2012)

Capacitor scaling

- Capacitance (parallel plate) $C = \dfrac{\varepsilon\varepsilon_0 A}{d} \quad \propto L$
- Charge $Q = CV = \varepsilon\varepsilon_0 \dfrac{A}{d} V$
- For V=constant $Q \propto \dfrac{A}{d}$ but for E=constant $Q \propto A$
- in good insulators $E_{breakdown}$ = 1 to 3 V/nm, but $E_{breakdown}$ can be 1000x lower in air
- *A* (surface area) is often the only effective dimensional parameter because *d* is limited by E_{BD}



C: capacitance
A: area
d: gap
V: voltage
E: electric field
E_{BD}: breakdown electric field
Q: charge
ε_0: permittivity of free space
ε or ε_r: relative permittivity

Voltage fluctuations in a capacitor

$\overline{E}_{th} = \frac{1}{2} k_B T = \frac{1}{2} C v_n{}^2$ Equi-partition theorem, like for noise in a resistor or in mechanical systems

$v_{n\,rms} = \sqrt{\frac{k_b T}{C}}$ $v_{n\,rms} = \sqrt{\frac{k_b T}{\varepsilon \varepsilon_0}} \sqrt{\frac{d}{A}}$ $\propto L^{-1/2}$ $\Delta Q_{rms} = C \cdot v_{n,rms} = \sqrt{k_b T C}$

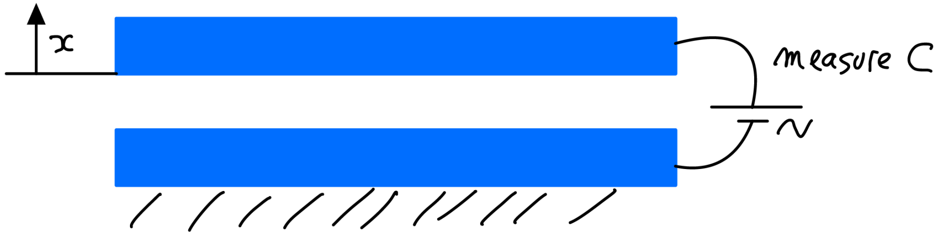
At room temperature (300K) $k_B T$ = 4.14 10^{-21} J

for discrete capacitor C=20 pF v_{rms}= 14 µV (i.e. 1000 e)

for 1 µm^2, 0.1 µm dielectric (ε=2) C=0.16 fF, v_{rms}= 5 mV, (i.e. 5e)

Capacitance sensing

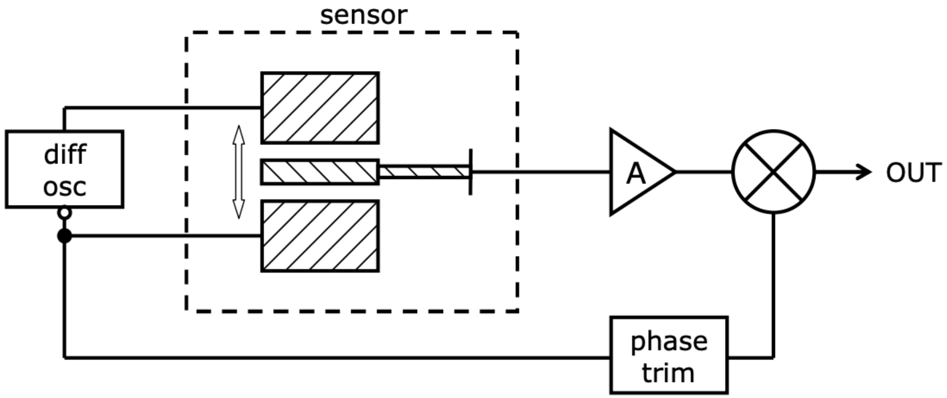
- Parallel plate capacitance with displacement x $\quad C = \dfrac{\varepsilon_0 A}{d+x}$
- Sensitivity $\quad S_0 = \dfrac{dC}{dx} = -\dfrac{C}{d} \qquad \boxed{S_0 \propto \dfrac{A}{d^2}} \propto L^0$
- Must scale down *d* to maintain S_0 if one reduces A

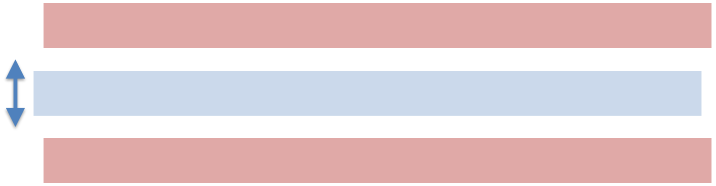


- Limits to downscaling capacitive sensing:
  - voltage noise
  - in small gaps, bias measurement voltage must be decreased due to E-field limitation
    => decrease of voltage sensitivity
  - defects / inhomogeneities in small gaps
  - probe voltage induce electrostatic force that could provoke pull-in
  - electrostatic "spring constant" affects dynamical properties $\quad k_{es} = \dfrac{dF_{es}}{dx}$

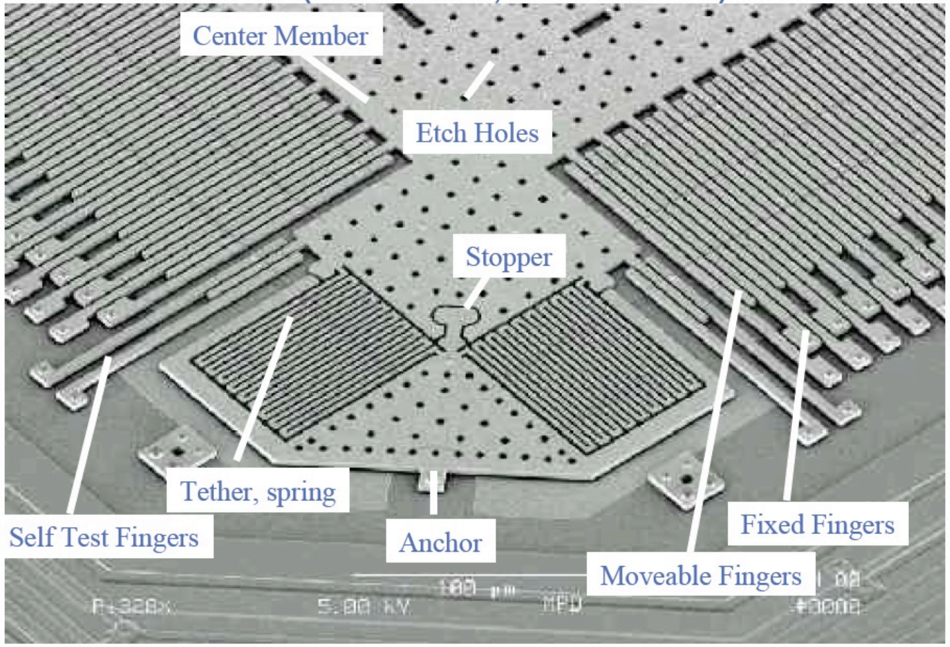
Capacitive sensing in MEMS

- Generally accomplished using a differential setup for accelerometers and gyros





Accelerometer beam (ADXL202, one corner)



(looks like a comb drive, but is not: fingers move perpendicular to their long axis, using many fingers to increase capacitance)

PARALLEL PLATE ELECTROSTATIC ACTUATORS

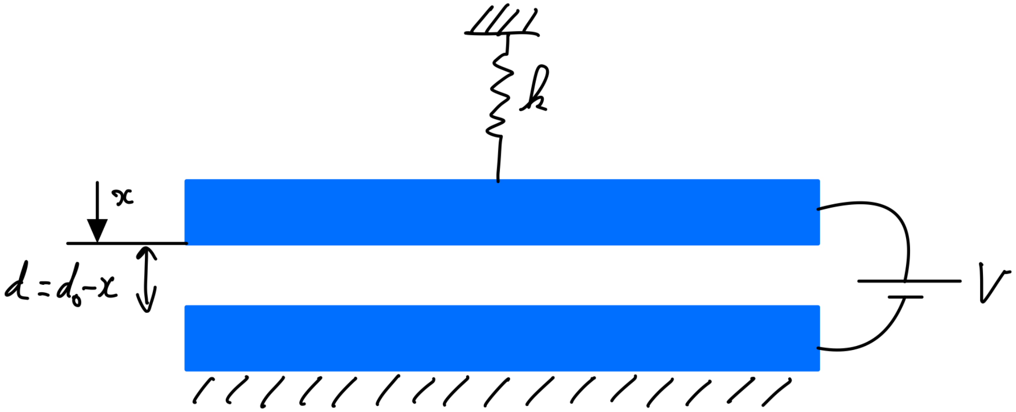
- Energy density, Paschen curve
- In-plane motion (constant gap)
- Closing gap motion
- Elastomer dielectric
- Zipping

Parallel plates electrostatic actuator

- Electrostatic energy in a capacitor $E_{es} = \frac{1}{2} C V^2$
- Normal electrostatic force for **fixed applied voltage V:**

$$F_{es} = \frac{dE_{es}}{dx} = \frac{1}{2}\frac{dC}{dx}V^2 = \frac{1}{2}\frac{CV^2}{d} \quad \propto 1/d^2$$

$$\boxed{F_{es} = \frac{\varepsilon_0 A V^2}{2d^2} = \frac{\varepsilon_0 A}{2} E^2}$$



- Or for **fixed charge Q**

$$F_{es} = \frac{Q^2}{2\varepsilon A}$$

(F independent of *d* for charge control!)

- Scaling of ES force:
  - **For constant Voltage (V indep of size):** $F_{ES} \propto L^0 \quad \propto \left(\frac{A}{d^2}\right)$
  - **For constant E field (V proportional to *d*):** $F_{ES} \propto L^2$

ES Force is always attractive!

Parallel plate electrostatic actuator

- Normal electrostatic force for an applied voltage V:

$$\boxed{F_{es} = \frac{\varepsilon_0 A V^2}{2d^2} = \frac{\varepsilon_0 A}{2} E^2}$$

- Energy density

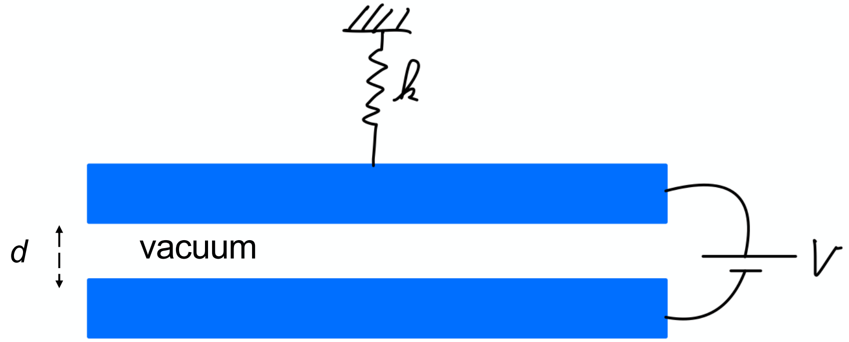
$$\boxed{w_{ES} = \frac{1}{2}\varepsilon_0 E^2}$$

- If add a dielectric

$$w_{ES} = \frac{1}{2}\varepsilon_0 \varepsilon_r E^2$$

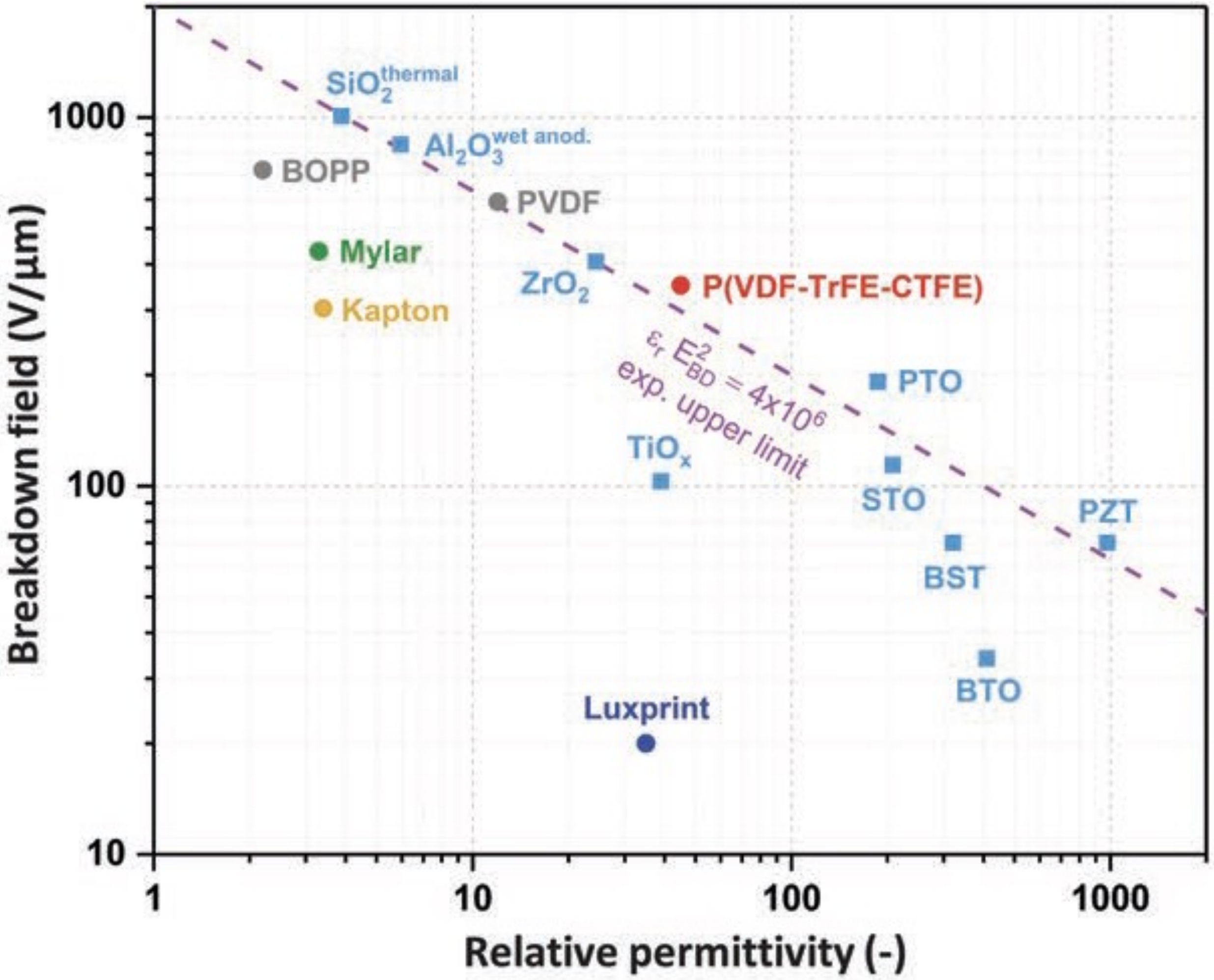
$$F_{ES} = \frac{1}{2}\varepsilon_0 \varepsilon_r A E^2$$

- Maximum energy density is limited by breakdown field E_{BD}



E=electric field, *V*=voltage,
d=insulator thickness, *A*= electrode area

There is an empirical upper limit to $\varepsilon_r . E_{BD}^2$ product for solid dielectrics



- Want materials with high ε_r **and** high $E_{breakdown}$
- **in MEMS, air is generally the dielectric to allow for motion**
- **But can also have solid dielectrics**

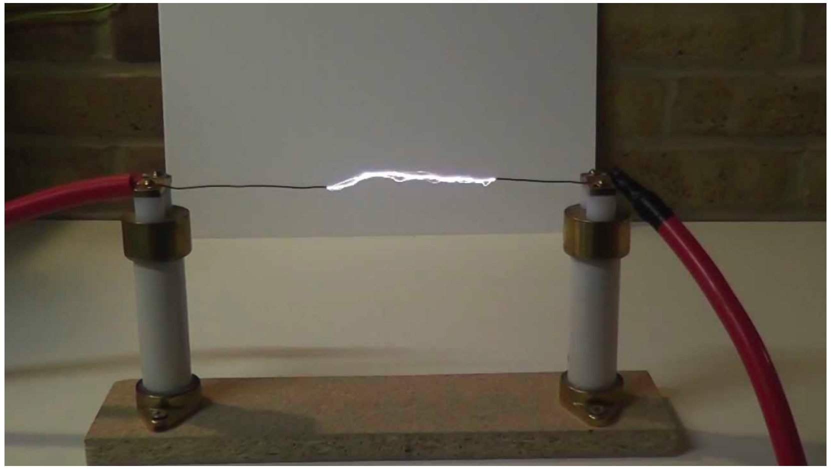
Adapted from Hinchet et al.
Advanced Materials Technologies **5**, 1900895 (2019)

Electrostatic actuation: energy density in air vs. in a solid

- Energy density $\boxed{w_{es} = \frac{1}{2}\varepsilon_0 E^2}$ $\quad w_{ES} = \frac{1}{2}\varepsilon_0 \varepsilon_r E^2$
- Maximum energy density is limited by breakdown field
- **in *air*,** for large gaps, $E_{max} \approx 10^6 V/m$

$$w_{\max} = \frac{1}{2}\varepsilon_0 E_{\max}^2 \cong 4.5\ J/m^3 = 0.045 mbar$$

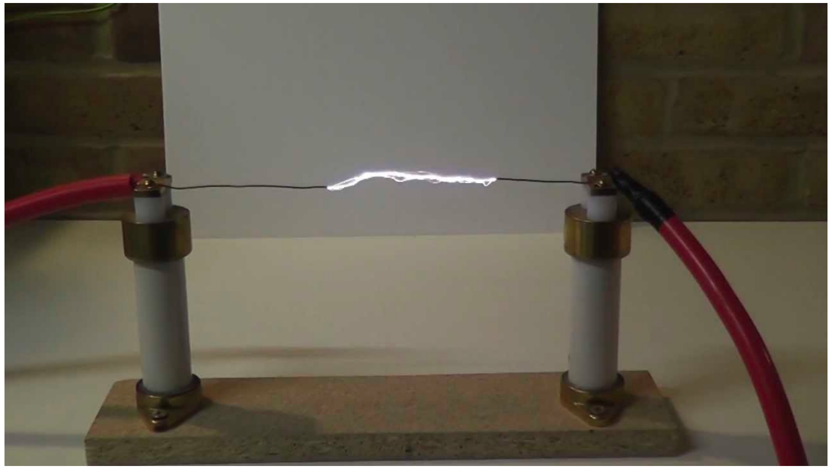
- But in **thin insulating films**, can have $E_{max} \approx 10^9 V/m$
  i.e. $w_{\max} = 45$ bar!



E: electric field
E_{ES}: electrostatic energy
w_{ES}: electrostatic energy density

Air (>50 µm gap) : 3.10^6 V/m, polycrystalline Al_2O_3: 2.10^7 V/m, thin film SiO_2: 1.10^9 V/m in MEMS/NEMS, plates are often separated by an air gap so they can move freely.

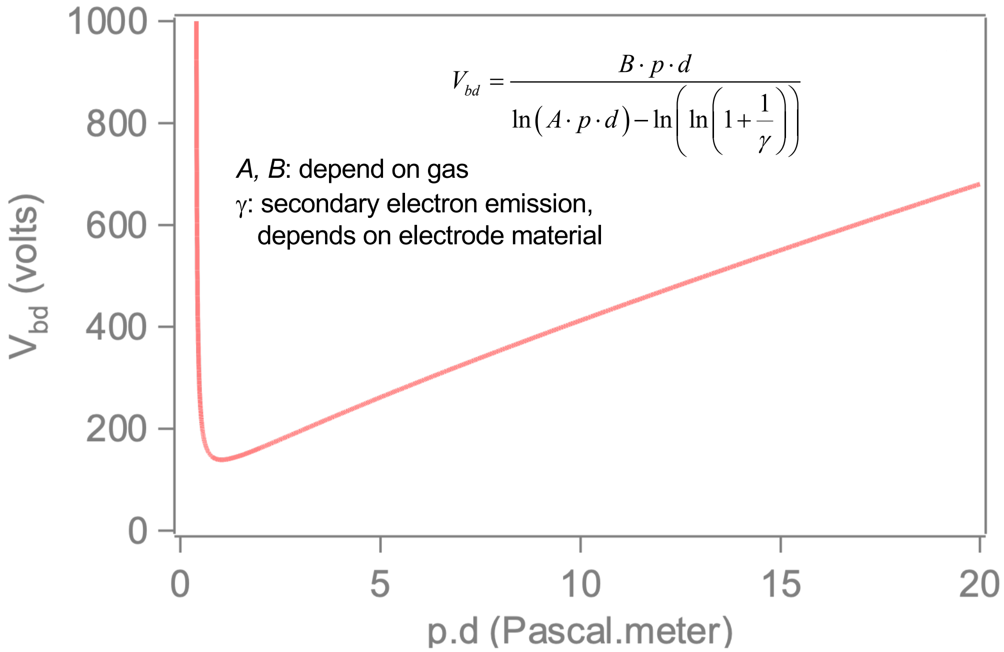
Why is the breakdown field lower in air than in solids?



https://youtu.be/MiHGDYeMAAU

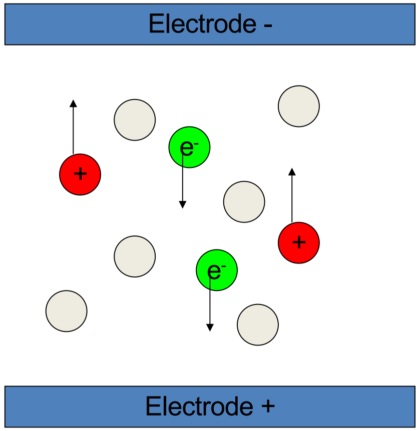
Sound comes from: “Tesla coil sparks. They are literally playing the music due to the programmed phase, pulse width and firing frequency! So, there are no speakers”

Paschen Curve: $V_{breakdown}$ vs. Pressure.distance



$$V_{bd} = \frac{B \cdot p \cdot d}{\ln(A \cdot p \cdot d) - \ln\left(\ln\left(1 + \frac{1}{\gamma}\right)\right)}$$

A, *B*: depend on gas
γ: secondary electron emission, depends on electrode material



Standard Paschen curve was derived for two large metal spheres, ignores field emission, fringing fields, etc.

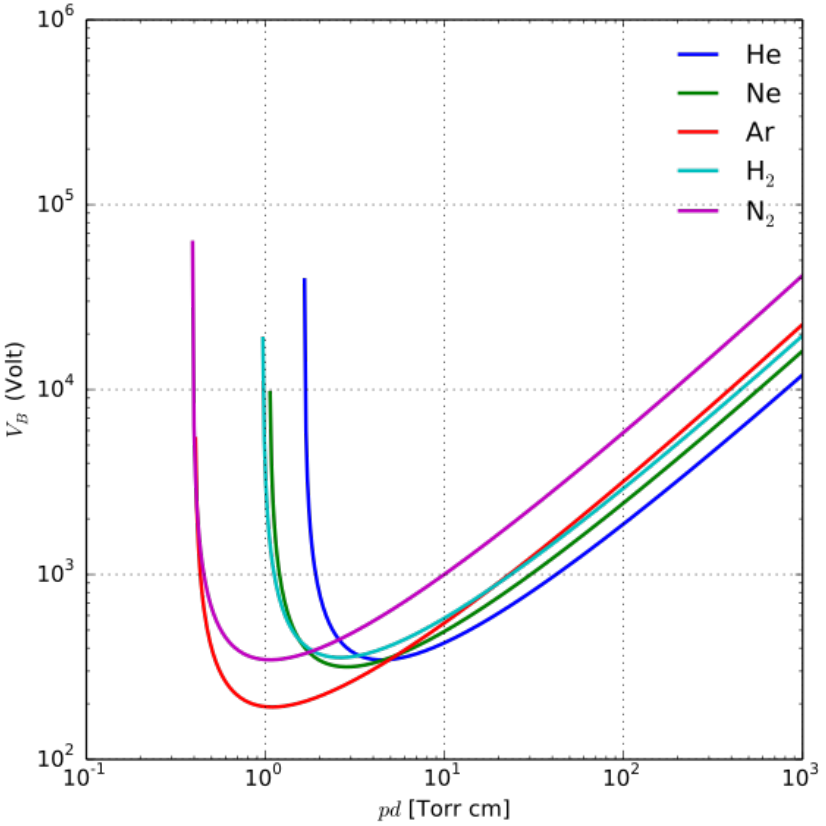
What is needed for ions to gain sufficient energy between collisions to ionize at impact and start avalanche?

Mean-free path is key

- At high P.d, there are sufficient gas molecules to allow for *Townsend avalanche breakdown* (ionization of gas by impact with electrons accelerated in electric field once they reach sufficient energy)
- At very low P.d, too few gas molecules to sustain avalanche: *vacuum isolation*

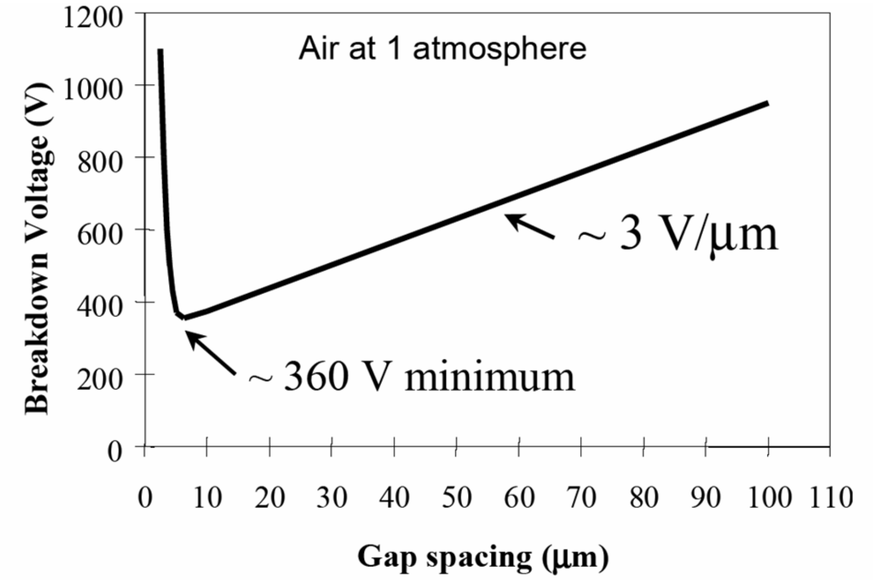
V_{bd}: breakdown voltage
p: gas pressure
d: gap between plates
A, B: gas-dependent constants

What is the maximum voltage we can apply in air at 1 atm?



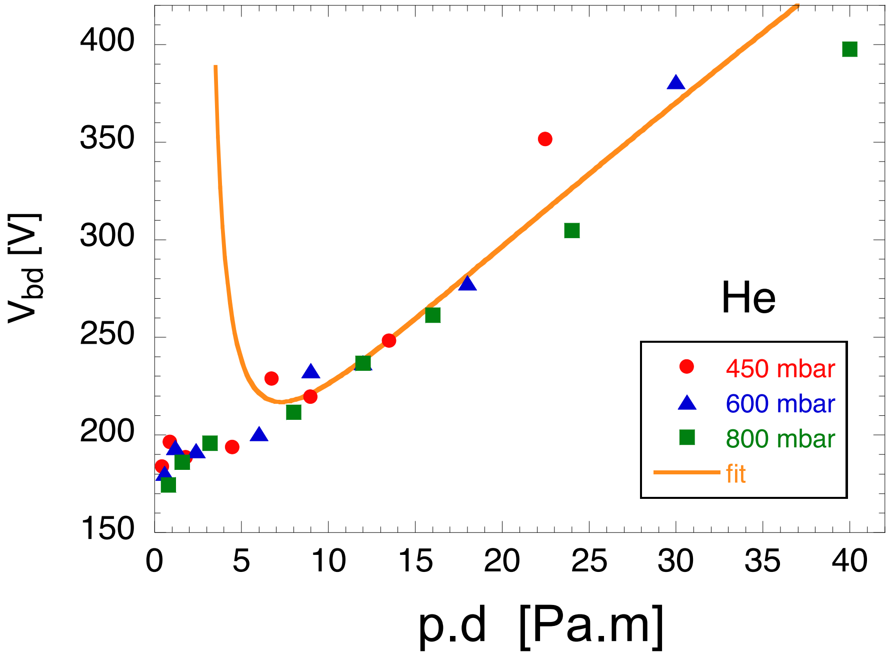
https://en.wikipedia.org/wiki/Paschen%27s_law

Theory (not reality!)

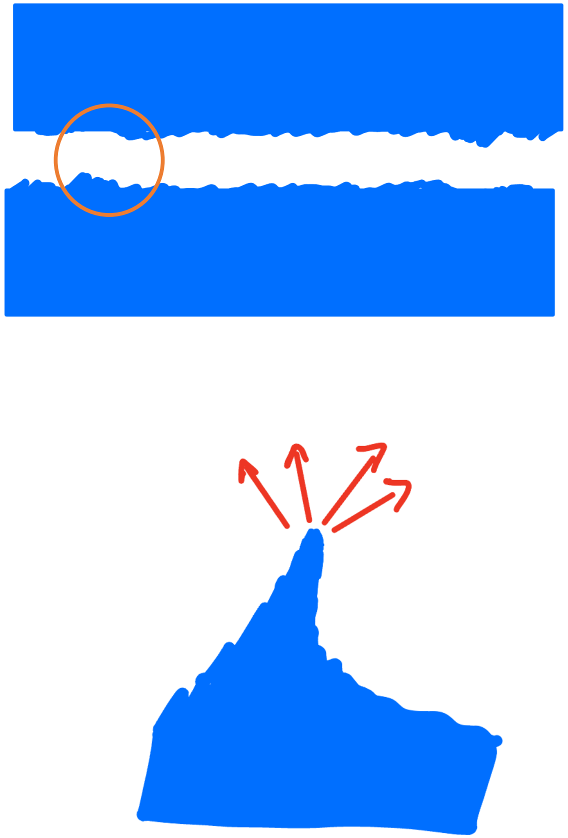


So then we never have breakdown if we operate below 360 V at 1 atm?

Measured Breakdown voltage for micromachined Aluminum electrodes (10 to 200 µm gaps)



(Carazzetti et al, SPIE Photonics West 2008)



- At scales of a few µm, we observe a breakdown V that decreases with decreasing gap
- This is mostly due to field emission of electrons

- 1 µm gap, 100 V: $E=10^8$ V/m
- But at surface asperities, get E field concentration, and $E>>10^9$ V/m

- A possible breakdown sequence:
  1. Electrons emitted
  2. Joule heating due to current
  3. Atoms evaporate due to heat
  4. Now the vacuum is full of atoms…
  5. … avalanche breakdown

Breakdown V in air at 1 atm

important

in air at 1 atm:

- At large distances, E-field for breakdown is constant at **3×10^6 V/m**
- Paschen theory indicates a steep increase of V_{max} at sub 10 µm gaps,
- But in small gaps (few µm), V_{max} is not observed to increase again.

Electrode spacing at minimum breakdown voltage (@ 1 bar):

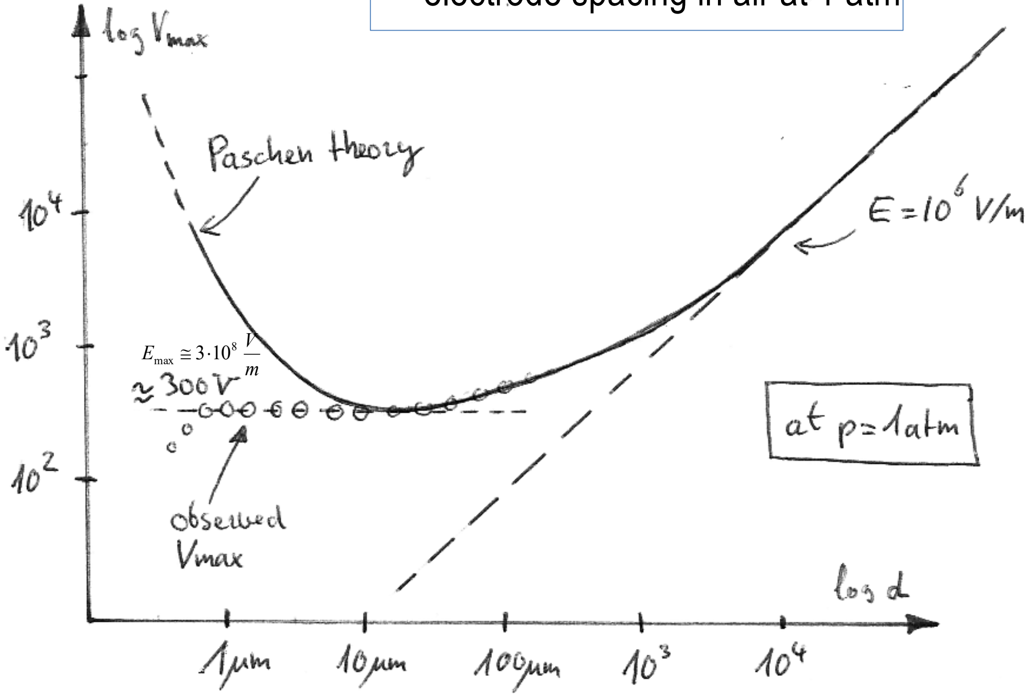
$h_{min} \cong$ 2-8µm and voltage at h_{min}:

$V_{min} \cong 300$V

For 1-5 µm gap, the breakdown voltage is around 300V and the maximum electrical field is typically:

$$E_{\max} \cong 3\cdot10^8\ \frac{V}{m}$$

breakdown Voltage vs. electrode spacing in air at 1 atm



Paschen curve leads to very high energy density!

- For 1 to 2 µm gap, maximum electrostatic energy density is

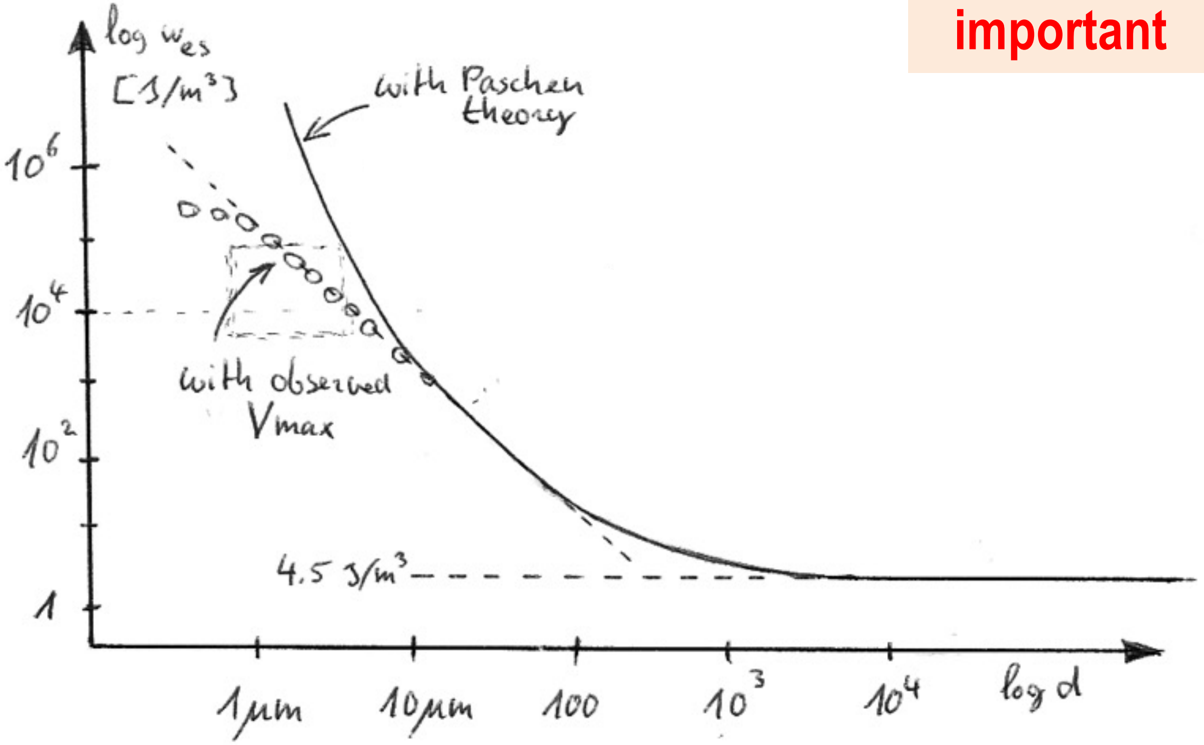
$$w_{\max} = \frac{1}{2}\varepsilon_0 E_{\max}{}^2 \cong 10^4 \; to \, 10^5 \quad J/m^3$$

0.1 to 1 bar

- Compared with magnetic actuators, ES energy density is higher for sub-10 µm gaps

Can get very high actuation pressures thanks to high E field in small air gaps

important



(rough) Comparison of energy densities between different actuation principles

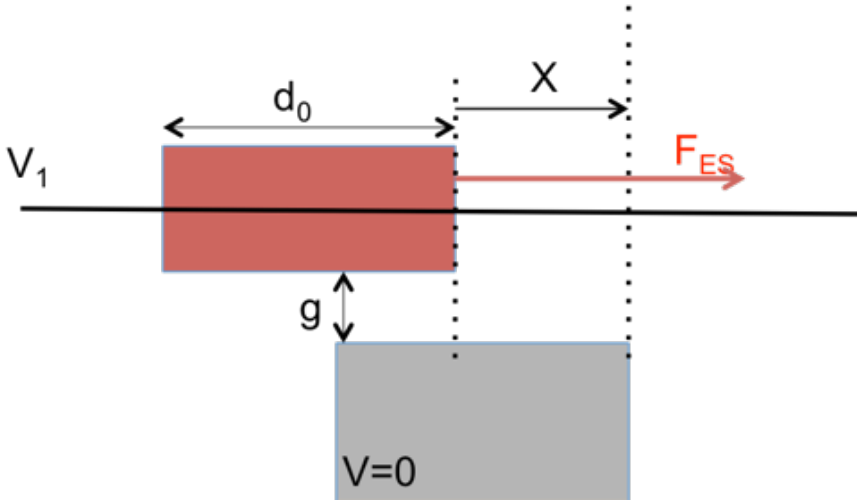
- Electrostatics (for small gaps) $w_{\max} = \frac{1}{2}\varepsilon_0 E_{\max}{}^2 \cong 10^4 \, to \, 10^5 \quad J/m^3$

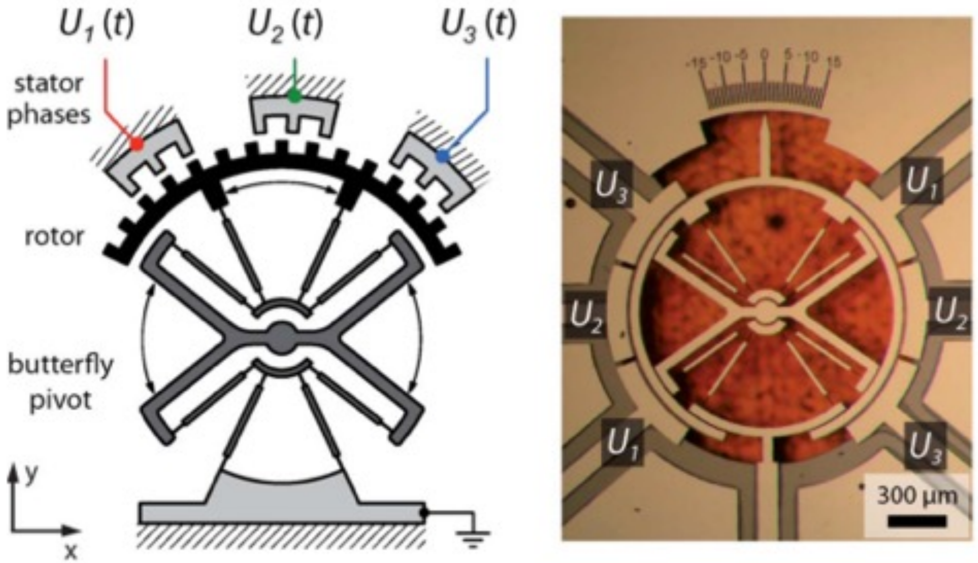
  (for large gaps) $w_{\max} \cong 10^1 \quad J/m^3$

- Magnetic (B_{sat} at 1T) $w_{\max} = \frac{1}{2\mu_0} B_{\max}{}^2 \cong 10^6 \quad J/m^3$ (for size > cm)
- Thermal Si (ΔT = 100° C) $w_{\max} = \frac{1}{2} Y\left(\alpha \cdot \Delta T\right)^2 \cong 5 \cdot 10^3 \quad J/m^3$
- Piezoelectric (E_{max}=30V/µm) $w_{\max} = \frac{1}{2} Y\left(d_{33} \cdot E_{\max}\right)^2 \cong 2 \cdot 10^2 \quad J/m^3$
- Pneumatic (p_{max} =1000 bar) $w_{\max} = p_{\max} \cong 10^8 \quad J/m^3$
- Mammalian Muscle $w_{\max} = p_{\max} \cong 10^6 \quad J/m^3$

Examples of ES actuator with constant gap (but varying overlap)

Electrostatic devices with fixed gap spacing (only change electrode overlap, not gap)





- red block at voltage V_1 can slide on a rail, at fixed gap *g* from grounded, fixed gray block,
- ES force “lines up” the two blocks.
- If have three offset positions, can make a stepper motor.

$$C = \varepsilon\varepsilon_0 \frac{(d_0 - x)t}{g}$$

$$\boxed{F = -\frac{dE}{dx} = -\frac{d}{dx}\left(\frac{1}{2}CV^2\right) = \frac{1}{2}\varepsilon\varepsilon_0 V^2 \frac{t}{g}}$$

Stranczl, M.; Sarajlic, E.; Fujita, H.; Gijs, M.A.M.; Yamahata, C., "High-Angular-Range Electrostatic Rotary Stepper Micromotors Fabricated With SOI Technology," JMEMS , vol.21,, pp.605 2012 doi: 10.1109/JMEMS.2012.2189367

Scaling: for constant V, $F \backsim L^0$, for constant E, then $F \backsim L^2$
(but we can’t reach the same *E* at large scale as for µm scale)

Electrostatic 3-phase Linear Stepper Motor Fabricated by Vertical Trench Isolation Technology

Edin Sarajlic, Christophe Yamahata,
Mauricio Cordero and Hiroyuki Fujita

© 2009/02, the University of Tokyo

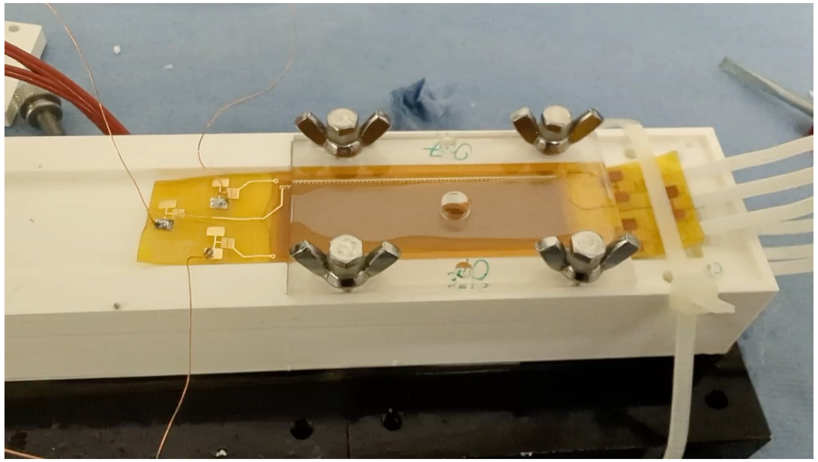
Stranczl, M.; Sarajlic, E.; Fujita, H.; Gijs, M.A.M.; Yamahata, C., JMEMS , vol.21, no.3, pp.605,620, 2012 doi: 10.1109/JMEMS.2012.2189367

MEMS in air

Macro-scale linear electrostatic motors

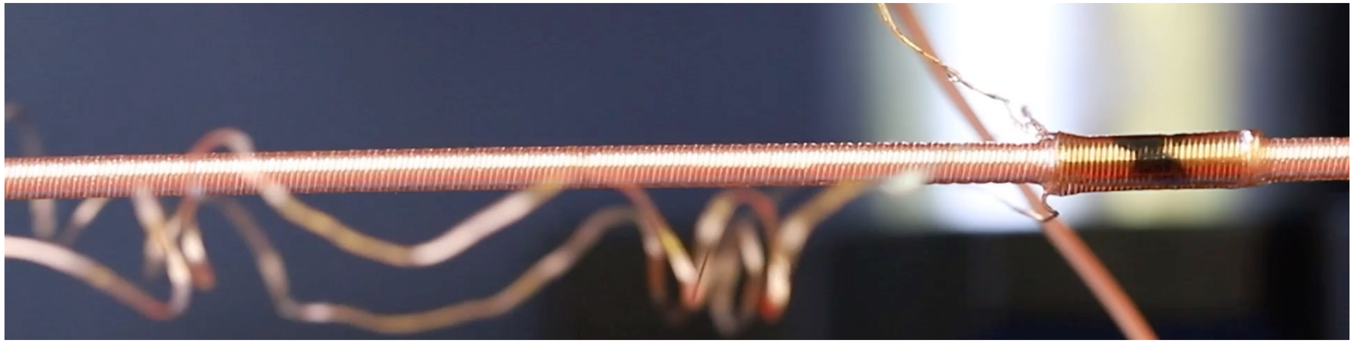
Electrode Gaps: 40-80 μm for high energy density
Immersed in oil for higher breakdown field

Ribbon Format



Martijn. Schouten

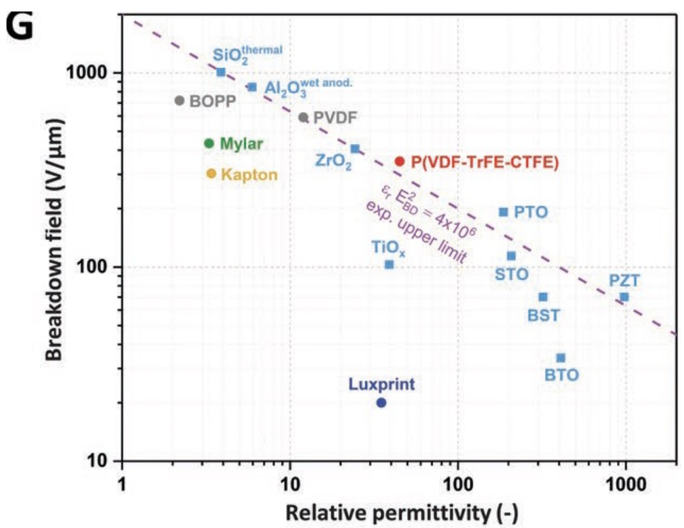
Fiber Format



Sylvain. Schaller

Unpublished, EPFL-LMTS

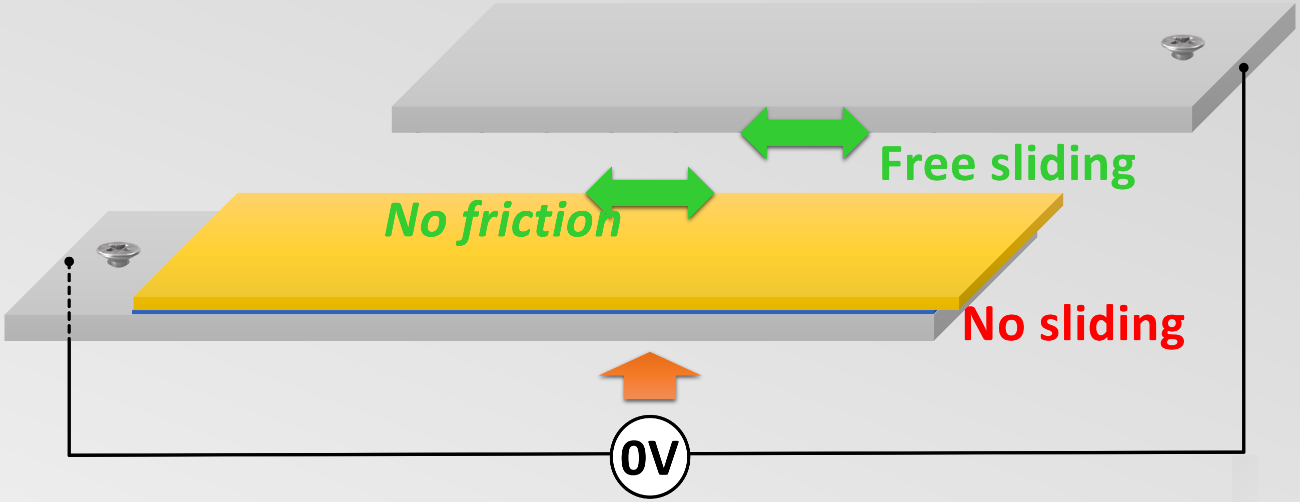
High-permittivity dielectrics for high ES forces

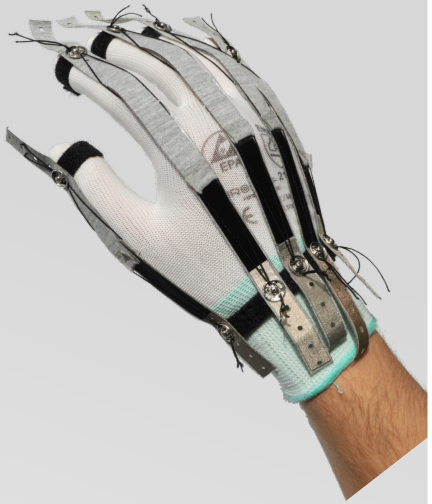


Haptic glove for VR + AR:
The clutch actively blocks finger motion to make virtual objects feel solid.

Hinchet and Shea, Adv. Mat. Tech. 2019
Hinchet and Shea, Adv. Intell. Sys. 2022

How does the ES clutch block motion?





No voltage

- ➢ Finger is free

Voltage on

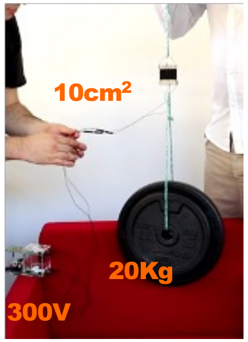
- ➢ Finger is blocked

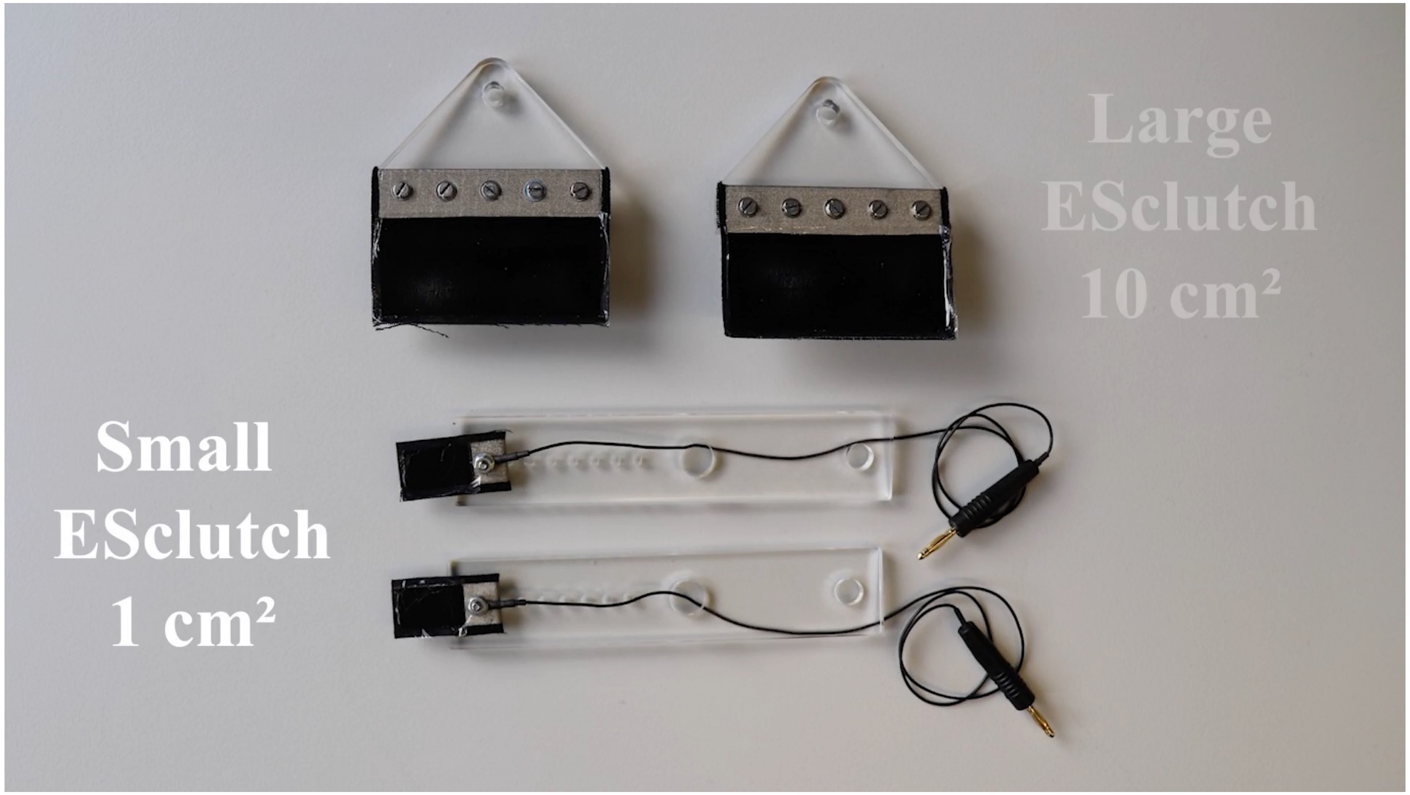
$$F_{\mathrm{F\,max}} = \mu \times F_{\mathrm{N\,max}} = \frac{\varepsilon_0 A}{2} \times \mu\, \varepsilon_r E_{\mathrm{bd}}^2$$

Textile ESclutch can block 2 kg/cm^2 at 300 V

Small ESclutch

Large ESclutch





- High holding force :
  20 N/cm^2 at 300 V
- low power ***1.2 mW/cm^2***
- Flexible, Lightweight ***30 mg/cm^2***
- Fast ***< 15 ms***

- Performance comes from use of ε_r=40 material, with E_{BD} >100 V/µm

- mW power enables use in in exoskeletons and full-body haptics
- Textile format

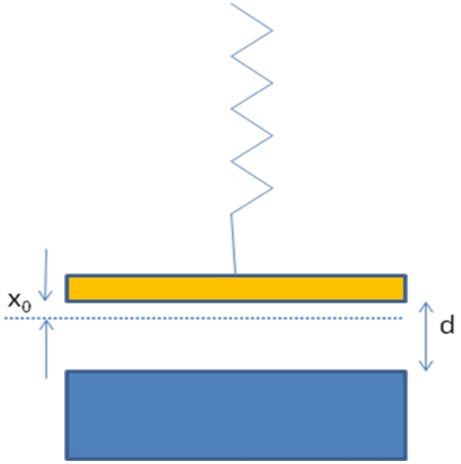
cm scale device, but 10 µm gap between electrodes…

Hinchet and Shea, Adv. Mat. Tech. 2019

Parallel plates electrostatic actuator. Gap changes

(electrodes move in direction of E field, no chnage in overlap)

1. Equilibrium position
2. Effective spring constant
3. Pull-in phenomena



Static equilibrium position for small displacements

$$\boxed{F_{elast} = F_{electrostatic}}$$

$$\boxed{k \cdot x_0 = \frac{\varepsilon_0 A V^2}{2\left(d - x_0\right)^2}}$$

Spring force

Electrostatic force

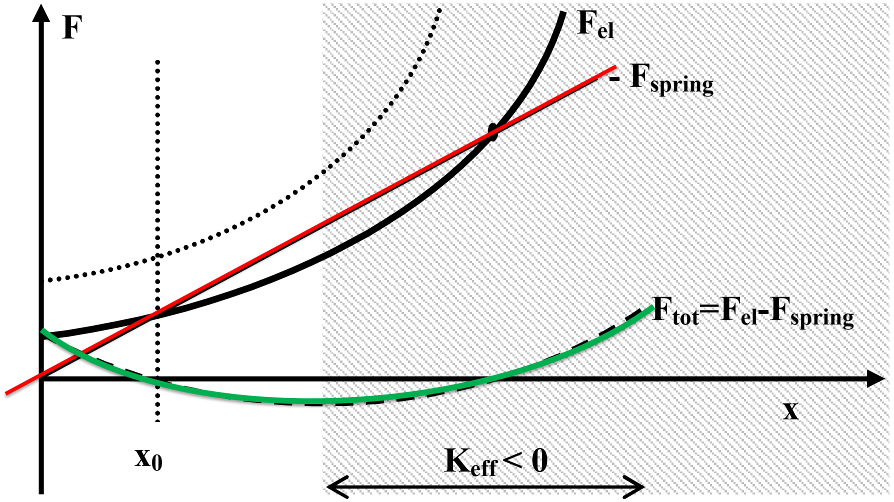
$$x_0 = \frac{\varepsilon_0 A V^2}{2kd^2} = \frac{Q^2}{2\varepsilon A k}$$

(Q= charge)

(only valid if x<<d)

Influence of electric field on effective spring constant

$$F(x) = -k_0 x + \frac{\varepsilon_0 A V^2}{2(d-x)^2}$$



(figure from S. Senturia's book)

- With non-linear electrostatic force, the effective system spring stiffness decreases
- Effective spring constant of system:

$$k_{eff} = -\frac{dF}{dx} = k - \frac{\varepsilon_0 A V^2}{(d-x)^3}$$

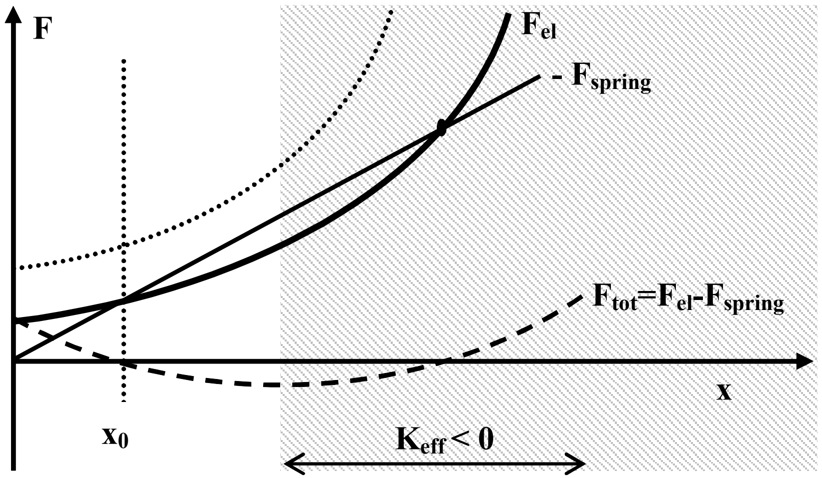
- With a bias voltage V, there is an apparent spring "softening" and thus a decrease of the natural frequency of oscillators:

$$\omega_{res}(V) = \sqrt{\frac{k_{eff}}{M}} = \sqrt{\frac{1}{M}\left(k - \frac{\varepsilon_0 A V^2}{(d-x)^3}\right)}$$

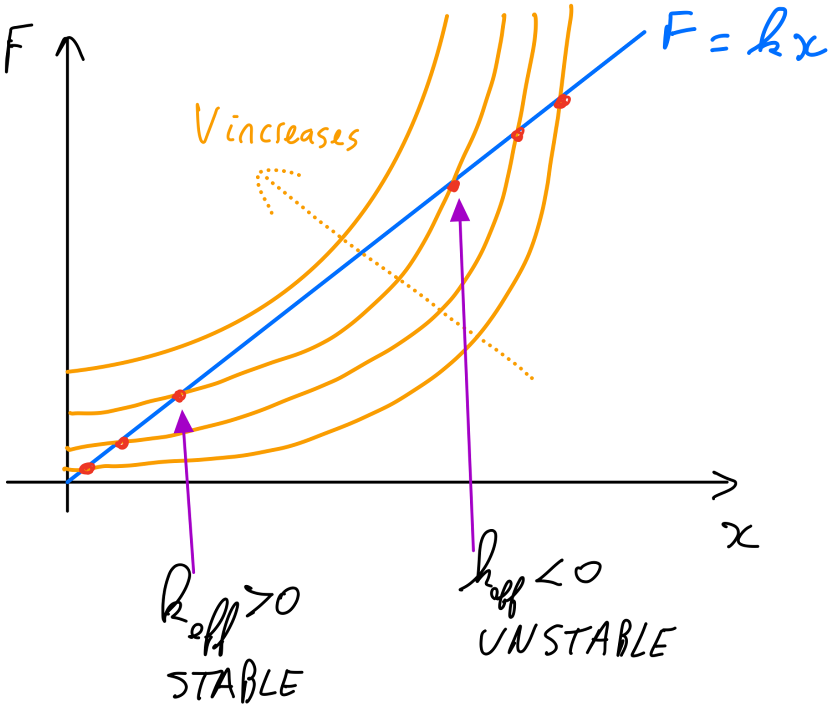
for small displacements $k_{eff} = k - \frac{k \cdot x_0}{d} = k\left(1 - \frac{x_0}{d}\right)$

- (good news) we can tune ω_0 by applying a bias voltage
- (bad news) we have an undesired shift in ω_0 at large displacements…

Pull-in voltage 1/2



(figure from S. Senturia's book)



$$F_{tot} = kx_0 + \frac{\varepsilon_0 A V^2}{2\left(d - x_0\right)^2} = 0$$

$$kx_c = \frac{\varepsilon_0 A V^2}{2\left(d - x_c\right)^2}$$

2 equilibrium positions
(but only 1 stable)

$$k_{eff} = -\frac{dF_{tot}}{dx} = k - \frac{\varepsilon_0 A V^2}{\left(d - x\right)^3}$$

Condition for stability: $k_{eff} > 0$

Pull-in voltage 2/2

Maximal stable position x_c when $k_{eff}\left(x_c\right)=0$

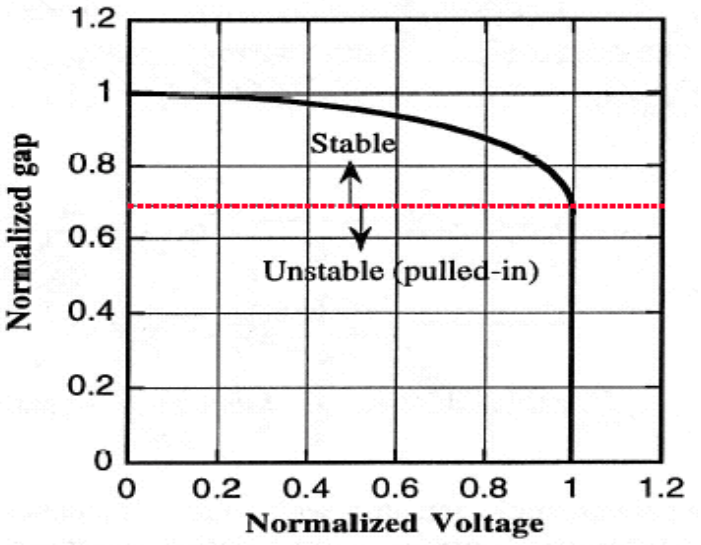
$$k=\frac{\varepsilon_0 A V^2}{\left(d-x_c\right)^3}=\frac{\varepsilon_0 A V^2}{2\left(d-x_c\right)^2}\frac{2}{\left(d-x_c\right)}=\frac{2kx_c}{\left(d-x_c\right)}$$

$$\boxed{x_c=\frac{1}{3}d}$$

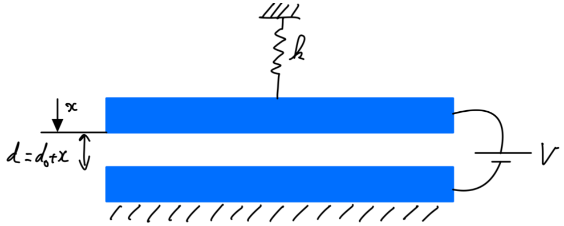
$$\boxed{V_{pull}=\sqrt{\frac{k}{\varepsilon_0 A}\left(\frac{2}{3}d\right)^3}}$$

$$V_{pull}\propto d^{3/2}$$

$$V_{pull}\propto L$$

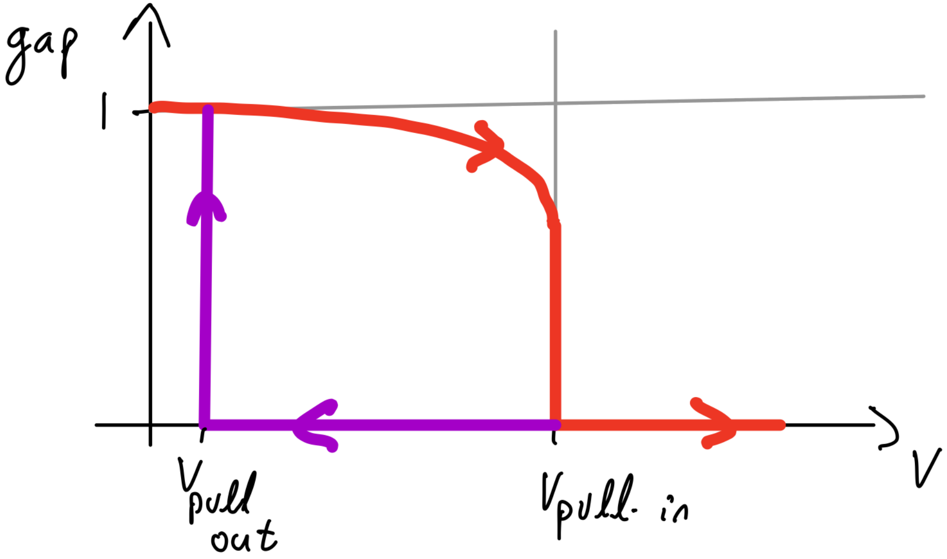


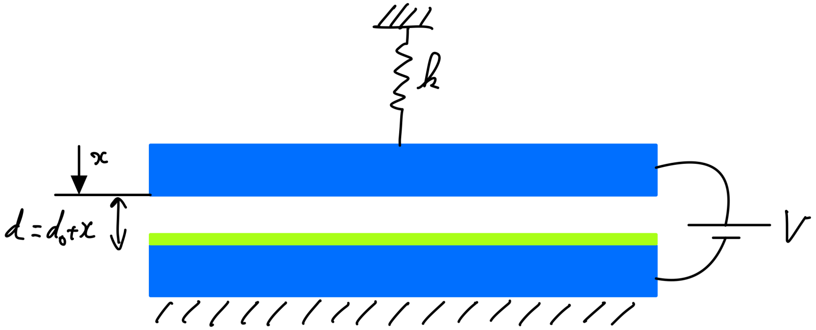
- **Only 1/3 of the gap can be used for actuation!**
- For doubly clamped beam, the pull-in limit can be up to ½ *h* because of non-linearity of mechanical restoring force.
- Stoppers or a dielectric film are needed to prevent short circuit when snapping in



$V_{\text{pull-in}}$ is a key design parameter for Electrostatic MEMS (spacing, size, shape…)

Pull in can be a desired feature (or a nuisance)



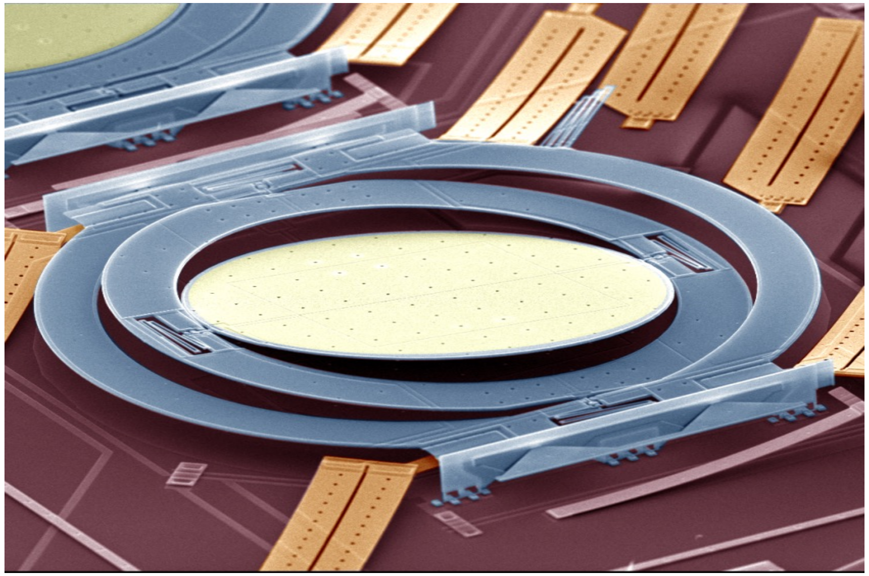


Hysteresis for pull-out (if have thin dielectric layer at bottom of gap to prevent a short circuit)

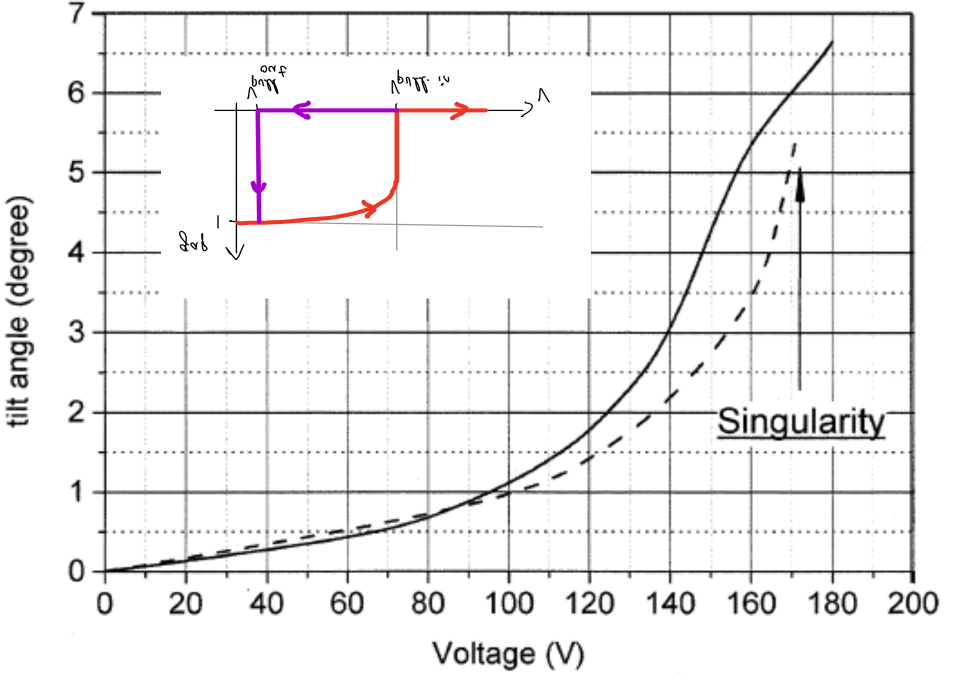
$$V_{pull-in} = \frac{2}{3}\sqrt{\frac{2kd^3}{3\varepsilon_0 A}}$$

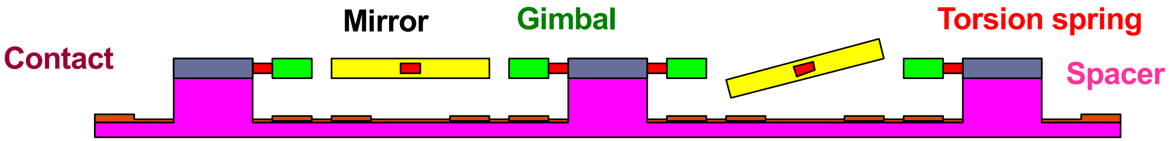
$$V_{pull-out} \sim t_{dielectric}\sqrt{\frac{2kd}{\varepsilon_{dielectric} A}}$$

Here pull-in is not desired, as it limits range over which angle can be controlled)



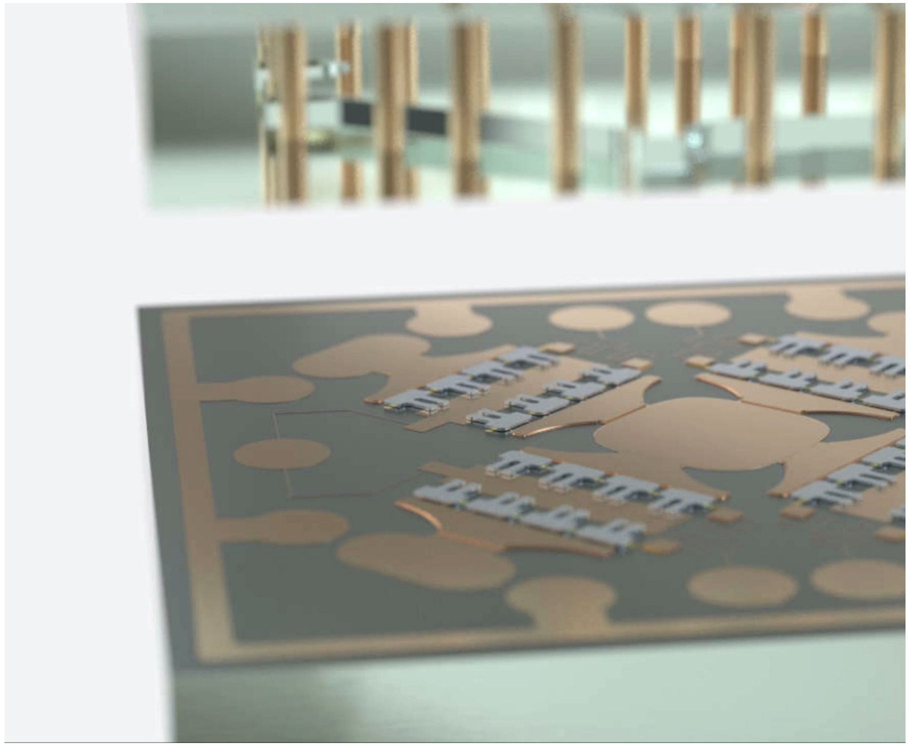
mechanical limit is about 12°, but can only access fraction





V. Aksyuk et al. (2003). Beam-steering micromirrors for large optical cross-connects. Journal of Lightwave Technology, 21(3), 634–642.

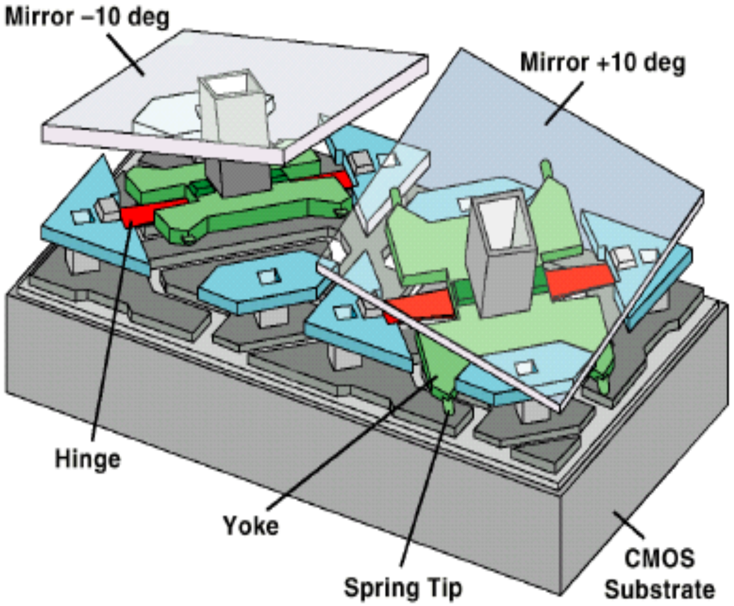
Electrostatic micro-relay (pull-in is a feature)

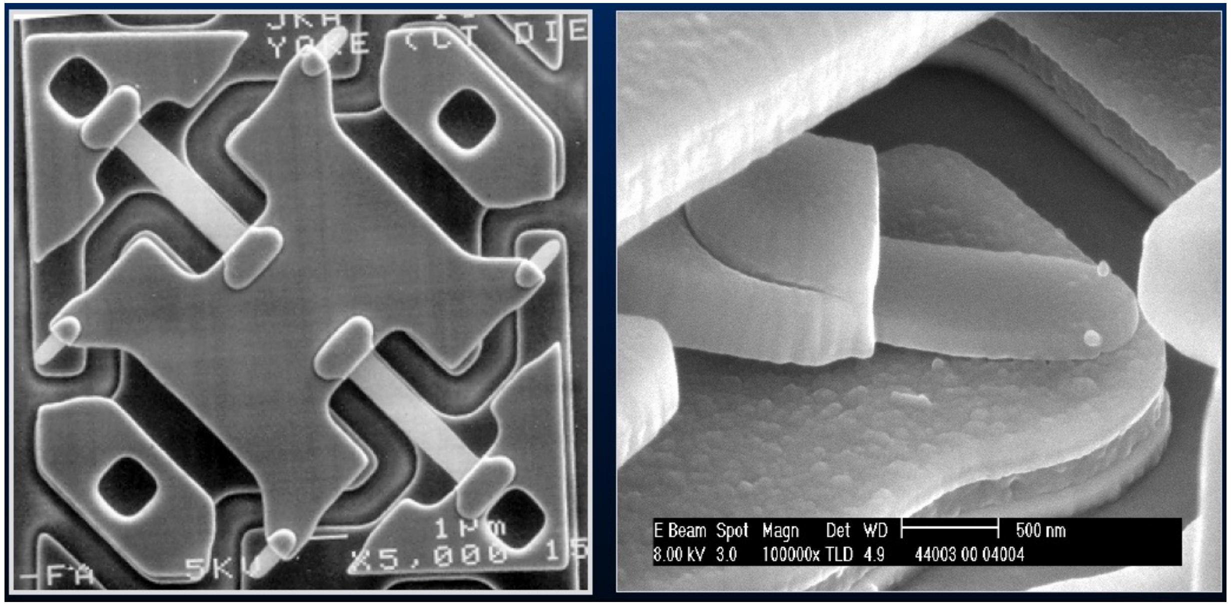


- Carries AC/DC and RF
- Linear performance from DC to > 50 GHz
- Switches at <10µs
- Performs reliably >3 Billion times
- Negligible resistance eliminates the need for bulky, heavy heatsinks

menlomicro

TI’s Digital MicroMirror devices: DMD. Relies on pull-in: does not need a precise voltage to get precise angle



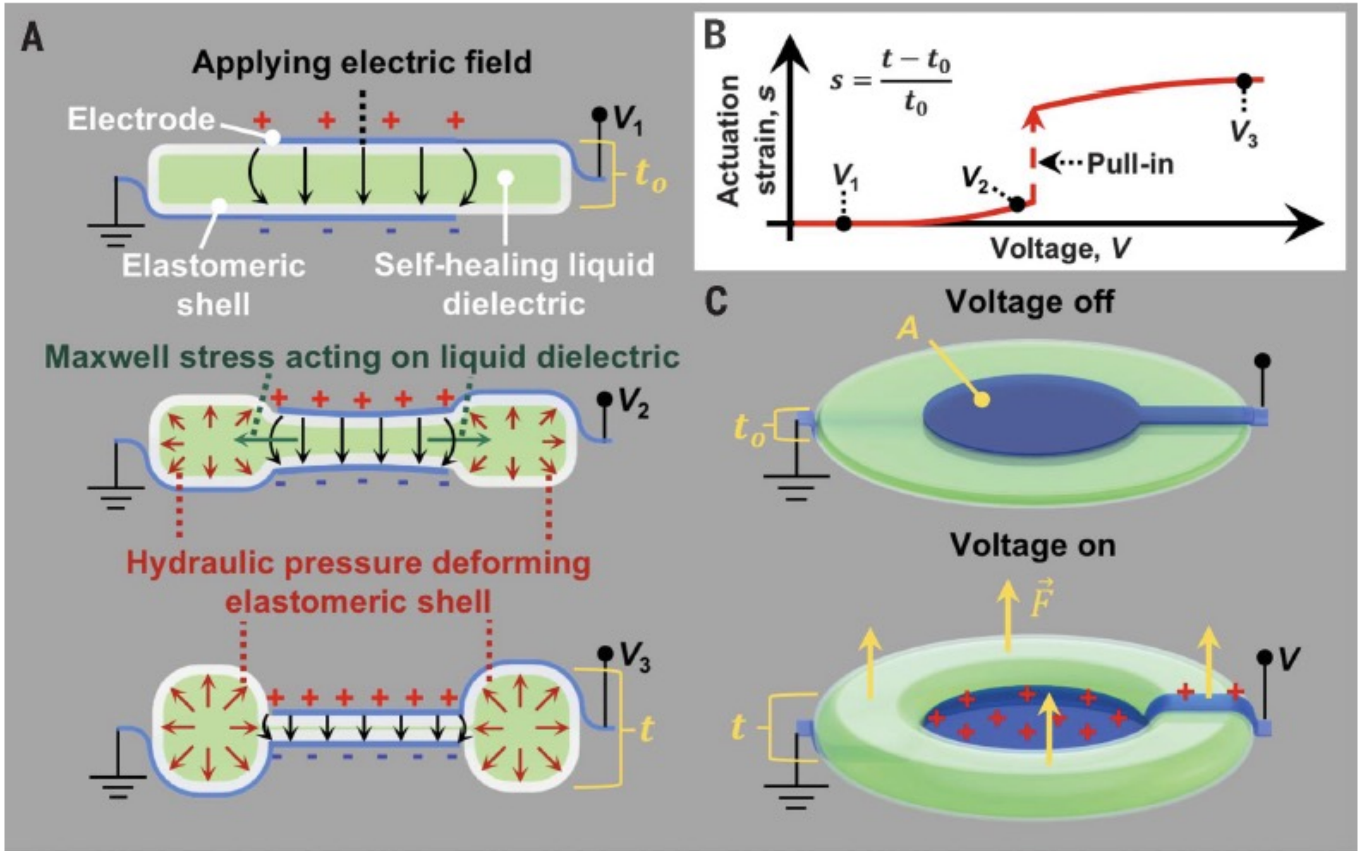


(pixel size approx. 10 µm x 10 µm)

- Underlying process is: CMOS + CMP planarization + Al-alloy mirror
- The hinge is only 60-100 nm thick (only 2-3 grains thick => no fatigue)
- Resonance frequency 50-200 kHz
- Anti-stiction PFDA self assembled monolayer
- **Gap of a few µm: can operate at 15-20 V, near max energy density from Paschen curve**

Need to address > 1 million pixels: need very high integration

Liquid dielectric: tolerant to breakdown, built-in hydraulic amplification

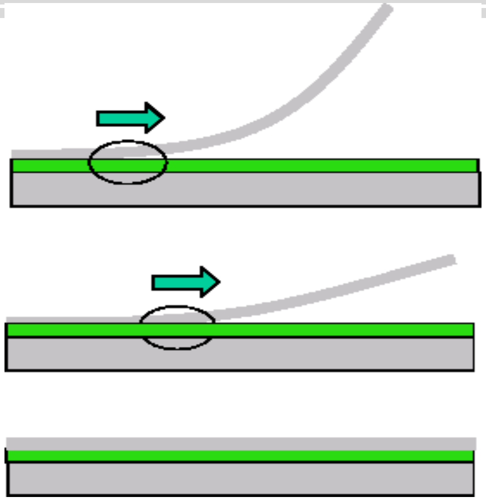


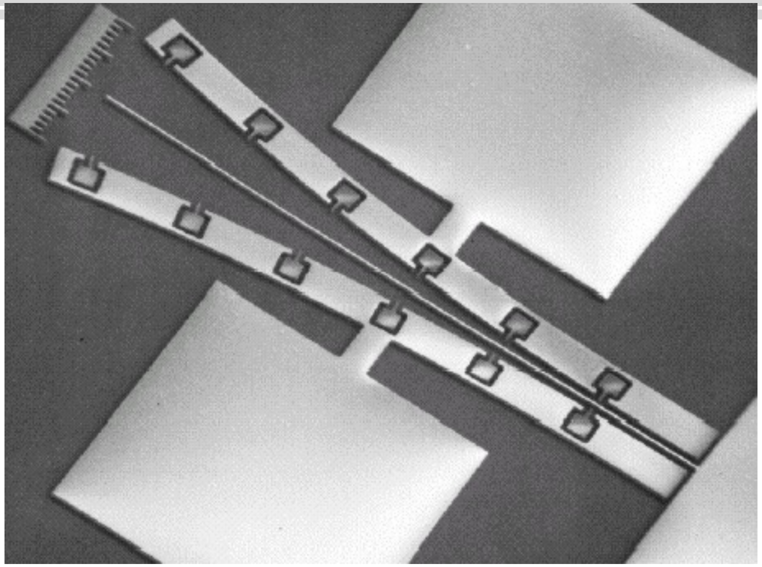
Acome, etal . "Hydraulically Amplified Self-Healing Electrostatic Actuators with Muscle-like Performance. " *Science* 359, no. 6371 (2018): 61–65. https://doi.org/10.1126/science.aao6139.

Electrostatic Zipping actuators are one way to:

- reduce voltage of electrostatic actuators
- while keeping large displacement
- Use materials with higher ε_r and higher E_{BD}

Zipping actuators, silicon MEMS





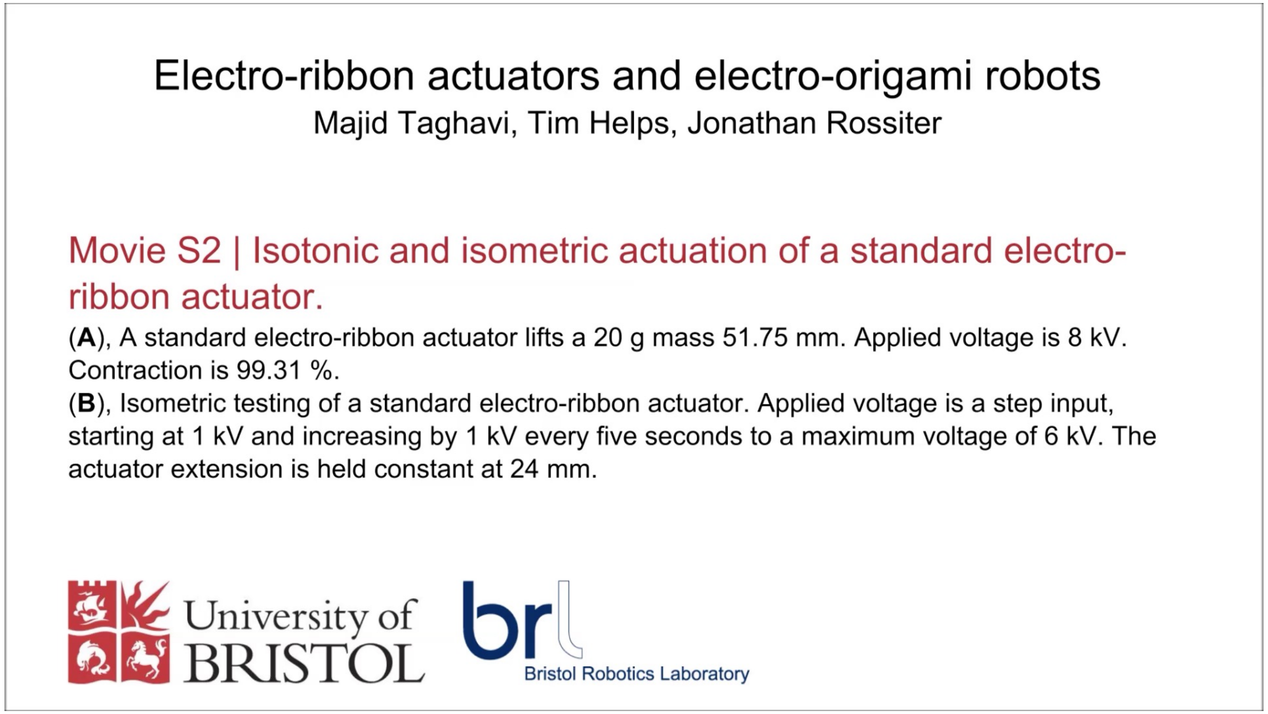
- High ES force generation, since always have a small gap
- Long-distance and stable displacement
- Possibly lower voltage drive (since small gap)

- Gain ES energy vs. Mechanical energy

- Limited use in MEMS due to stiction

J. Branebjerg, P. Gravesen, “A New Electrostatic Actuator providing improved Stroke length and Force”, MEMS‘92, Travemünde (Germany)

Electrostatic zipping devices, macro-scale (but µm insulator)



Electro-ribbon actuators and electro-origami robots

Majid Taghavi, Tim Helps, Jonathan Rossiter

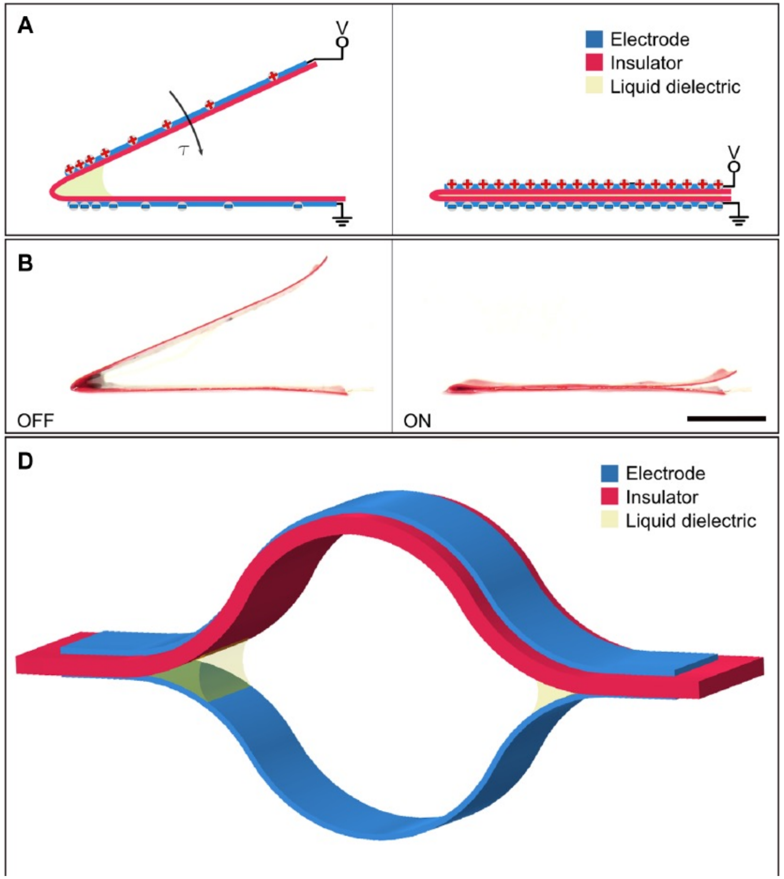
Movie S2 | Isotonic and isometric actuation of a standard electro-ribbon actuator.

(**A**), A standard electro-ribbon actuator lifts a 20 g mass 51.75 mm. Applied voltage is 8 kV. Contraction is 99.31 %.

(**B**), Isometric testing of a standard electro-ribbon actuator. Applied voltage is a step input, starting at 1 kV and increasing by 1 kV every five seconds to a maximum voltage of 6 kV. The actuator extension is held constant at 24 mm.

University of BRISTOL

brl Bristol Robotics Laboratory

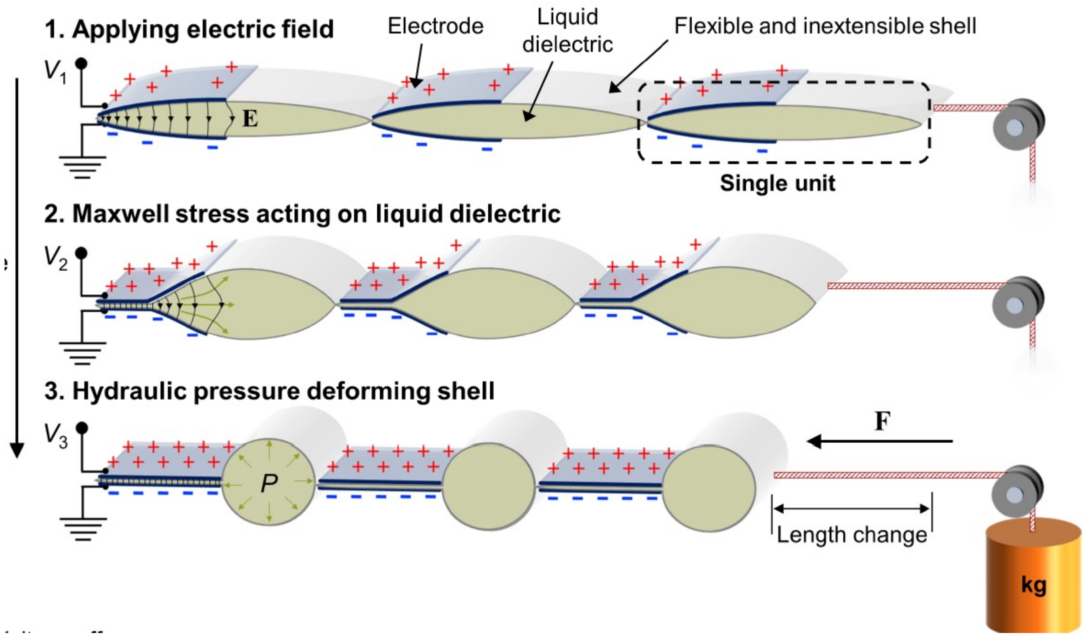


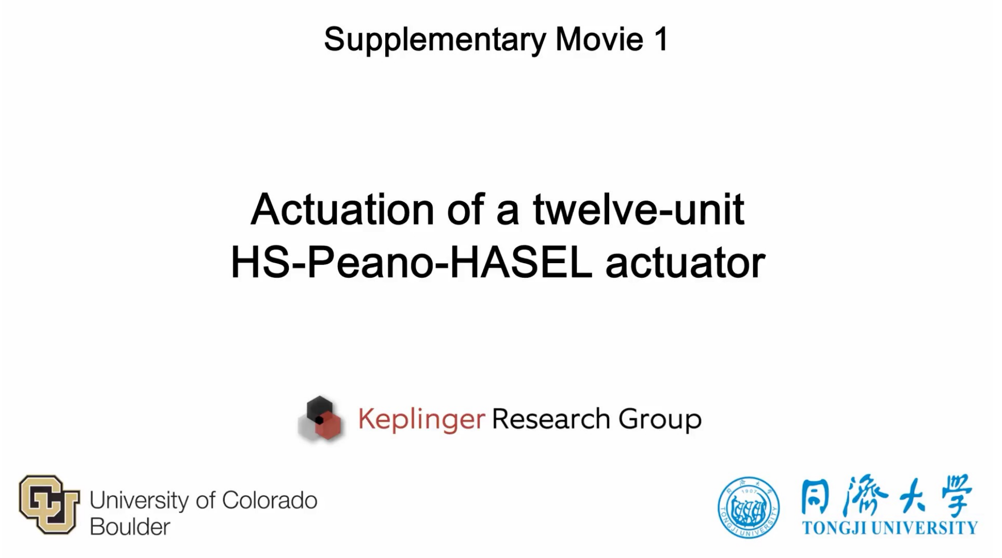
- Electro-ribbon actuators (Rossiter group)
- “Electro-origami principle [dielectrophoretic liquid zipping]”

M. Taghavi, T. Helps, J. Rossiter, Electro-ribbon actuators and electro-origami robots. *Science Robotics*. **3**, eaau9795 (2018).

Electrostatic zipping devices, macro-scale (but µm insulator)

- Peano-HASEL (Keplinger group)




Supplementary Movie 1

Actuation of a twelve-unit HS-Peano-HASEL actuator

Keplinger Research Group

University of Colorado Boulder

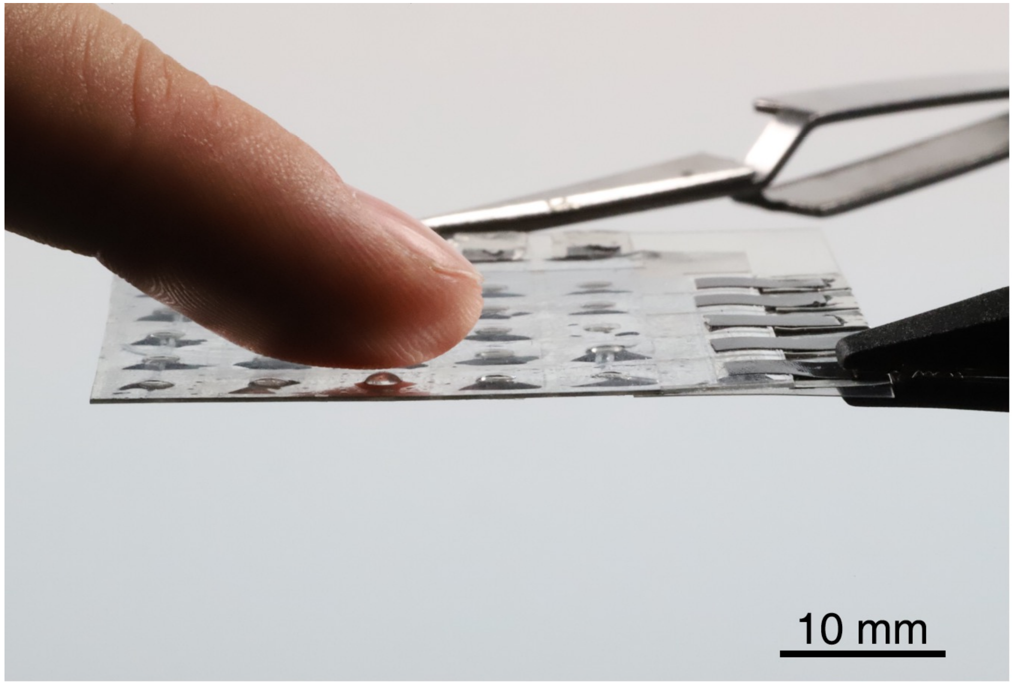
同濟大學 TONGJI UNIVERSITY

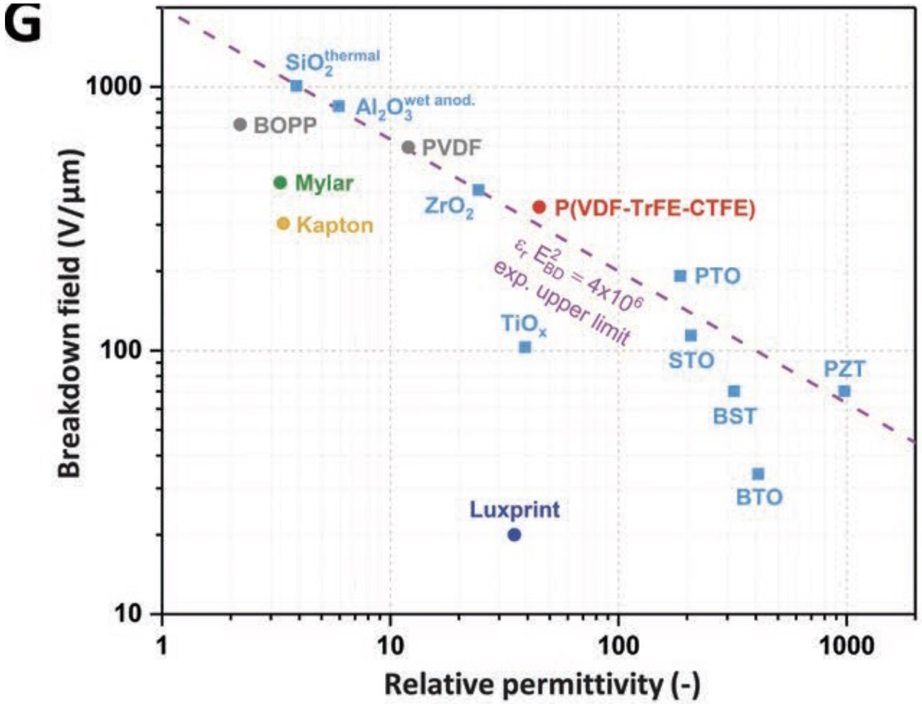
In all these zipping devices, the dielectric layer is 10-50 µm thick. Generally thickness not scaled down/up as area changes.

N. Kellaris, V. G. Venkata, G. M. Smith, S. K. Mitchell, C. Keplinger, Peano-HASEL actuators: Muscle-mimetic, electrohydraulic transducers that linearly contract on activation. *Science Robotics*. **3**, eaar3276 (2018).

Electrostatic zipping devices

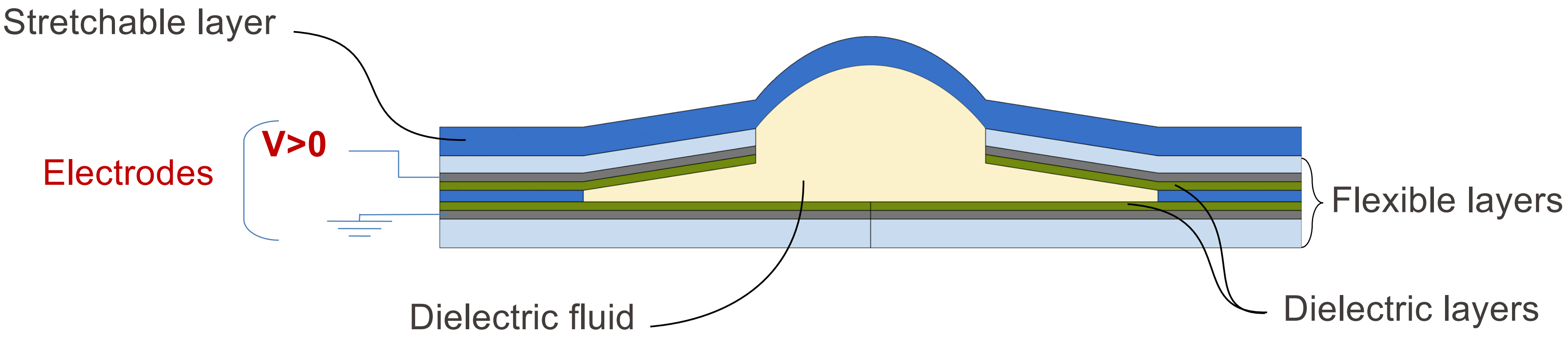
- HAXEL (EPFL-LMTS)





Leroy and Shea, Adv. Mat 2020
doi: 10.1002/adma.202002564

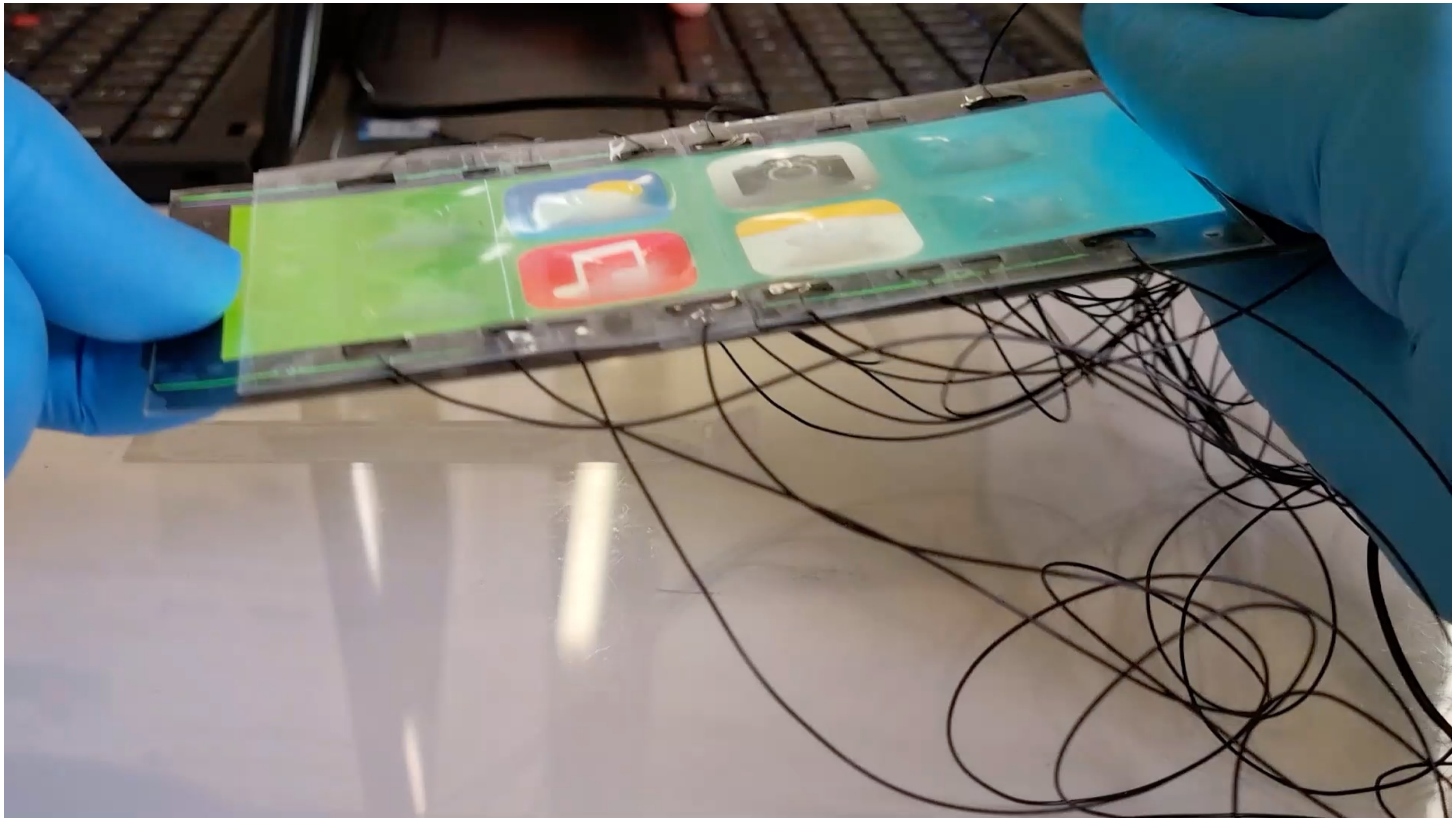
HAXELs: Hydraulically Amplified electrostatic taXELs



- Non-stretchable electrode and $\varepsilon_r=40$ dielectric
- Built-in hydraulic amplification
- But central stretchable silicone region

E. Leroy et al, Adv. Mat 2020

E. Leroy et al, Adv. Mat Tech 2023

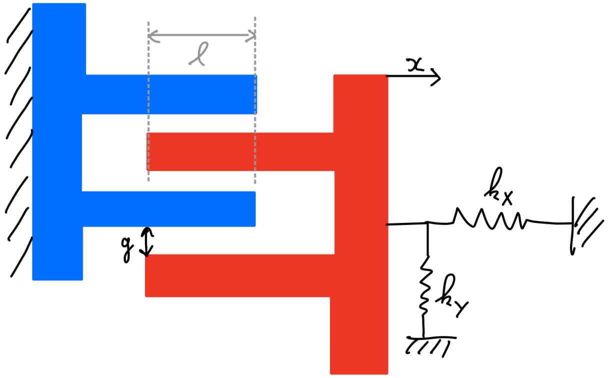


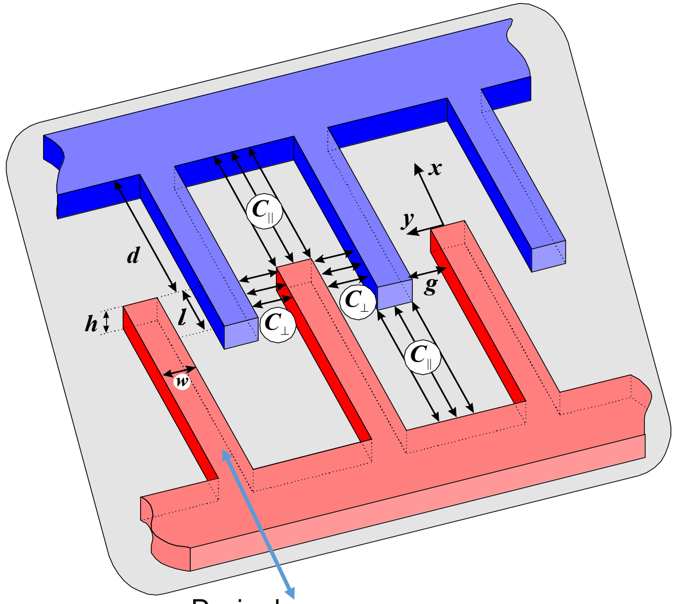
Firouzeh, Mizutani, Adv. Mat 2023

COMB DRIVE ACTUATORS

The «classic» MEMS actuator

Comb drive: overcomes many limitations of parallel plate MEMS





Desired motion direction

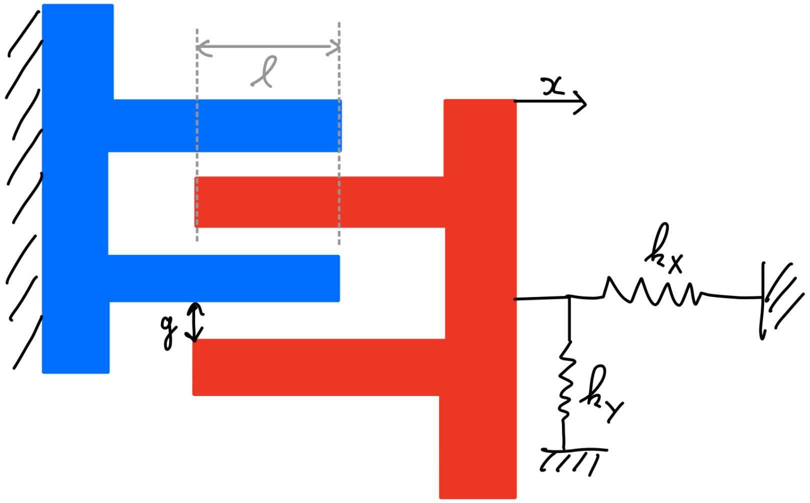
Capacitance $C(l) = 2N\varepsilon_0\left(\frac{l \cdot h}{g} + C_{par}\right)$

Longitudinal sensitivity: $\frac{dC}{dx} \cong N\frac{2\varepsilon_0 h}{g}$

$\Rightarrow$ does not depend on total overlap!!
$\Rightarrow$ No x dependence, so no instability

l: overlap length
h: height or thickness
g: gap
N: number of combs

Comb drive actuator



Axial force $$\boxed{F_x = \frac{1}{2}\frac{dC}{dx}V^2 = N\cdot\frac{\varepsilon_0 h}{g}V^2}$$

No x dependence!

Equilibrium Position $$\boxed{x = \frac{N\varepsilon_0 h\, V^2}{k_x\, g}}$$

Axial electrostatic Force is the same regardless of overlap! Depends on V^2

Effective spring constant = k_x because F_{es} has no x dependence

No spring softening for comb drive

(but if pull too far, k_x will not be linear, probably get parasitic y motion from flexure support)

Comb drive actuators

- Axial force is independent of overlap length

- The electrostatic force does not depend on displacement x
  => linear (no spring softening)

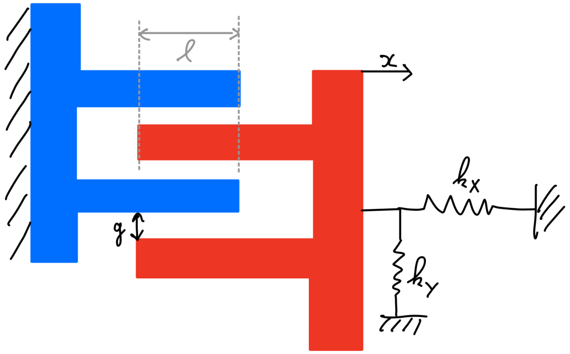
- Comb drive allow large displacement, but resonance frequency is limited by the spring mechanism

- Often smaller forces than parallel plate actuators (larger gap)

- The force can be increased by using high aspect-ratio structures: h >100 μm for SOI

Comb drive actuator

Lateral force $$F_y = \frac{1}{2} N \varepsilon_0 V^2 hl \left(\frac{1}{(g-y)^2} - \frac{1}{(g+y)^2} \right)$$

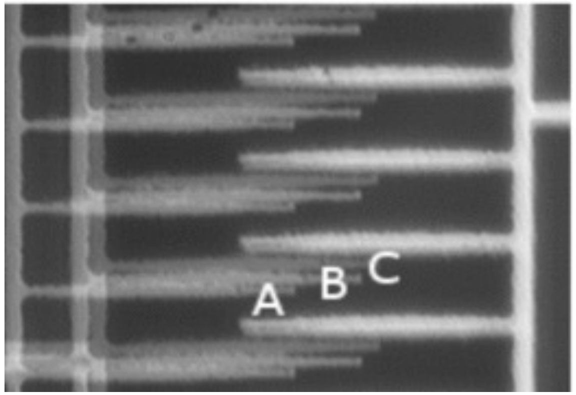


= 0 but only as long as y=0 …

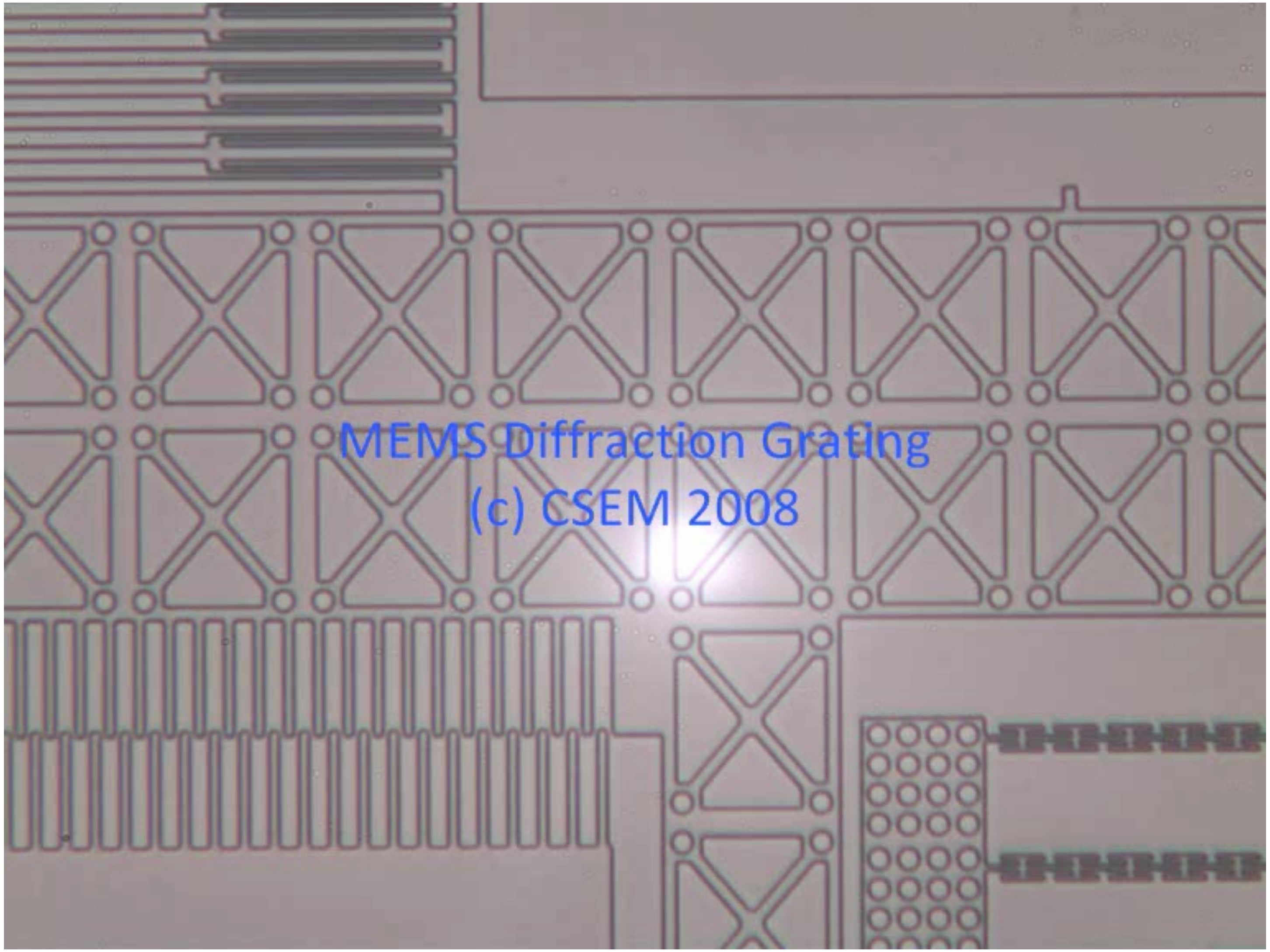
Transverse stability (lateral pull-in)

Stability condition (positive spring constant)

$$k_y > \left. \frac{dF_y}{dy} \right|_{y=0} \quad \text{or} \quad \frac{k_y}{k_x} > 2 \frac{x_{\max}(l + x_{\max})}{g^2}$$

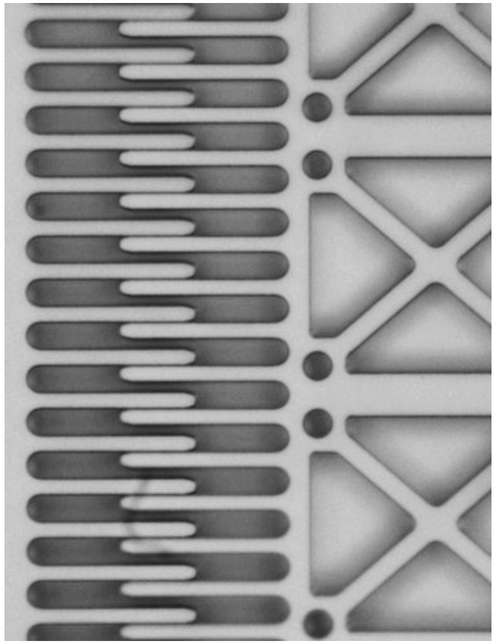


“The lateral instability problem in electrostatic comb drive actuators: modeling and feedback control”
B Borovic et al, (2006) *Journal of Micromechanics and Microengineering*, Volume 16, Number 7

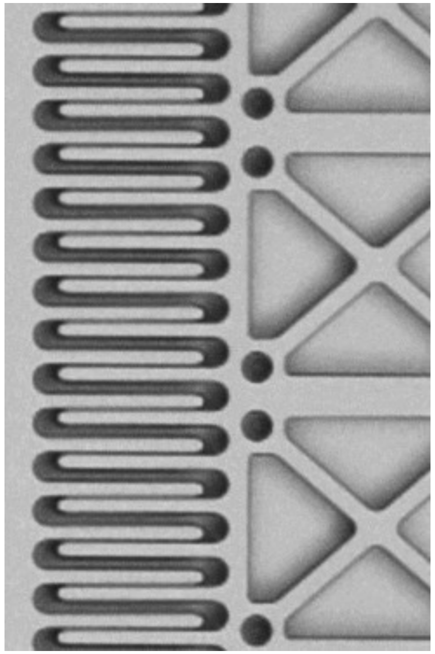


MEMS Diffraction Grating

(c) CSEM 2008



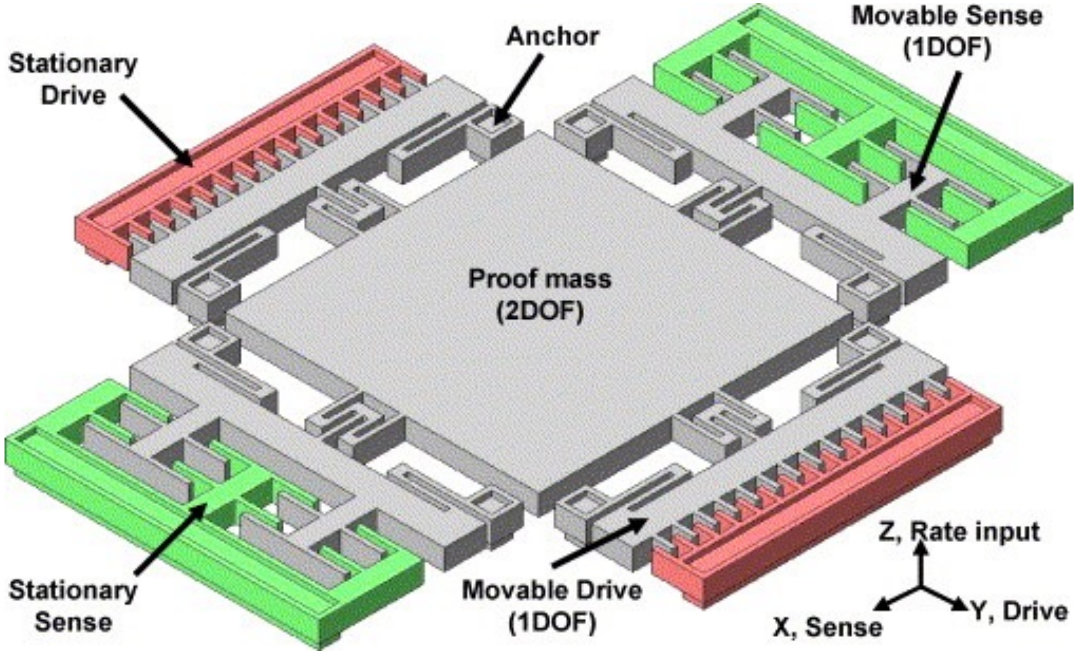
Typical overlap at snap-in

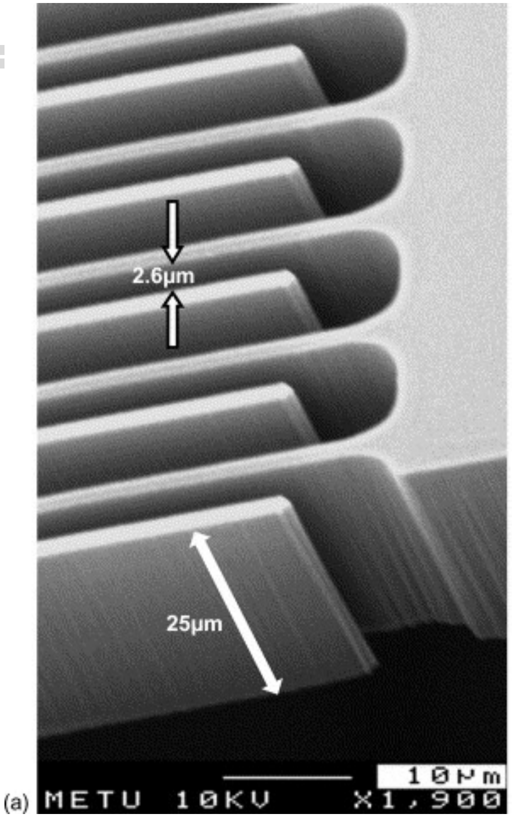


Lateral snap-in with shock

S. Sundaram, et al, *Journal of Micromechanics and Microengineering*, 21, p.045022 (2011)

Example of a MEMS gyro





Typical 1:10 aspect ratio for gap /depth
(SOIMUMPS)
http://www.memscap.com/products/mumps/soimumps/

Alper, et al. (2007). “A high-performance silicon-on-insulator MEMS gyroscope operating at atmospheric pressure”. *Sensors and Actuators, A: Physical*, *135*(1), 34. http://doi.org/10.1016/j.sna.2006.06.043

Electrostatic spring softening example

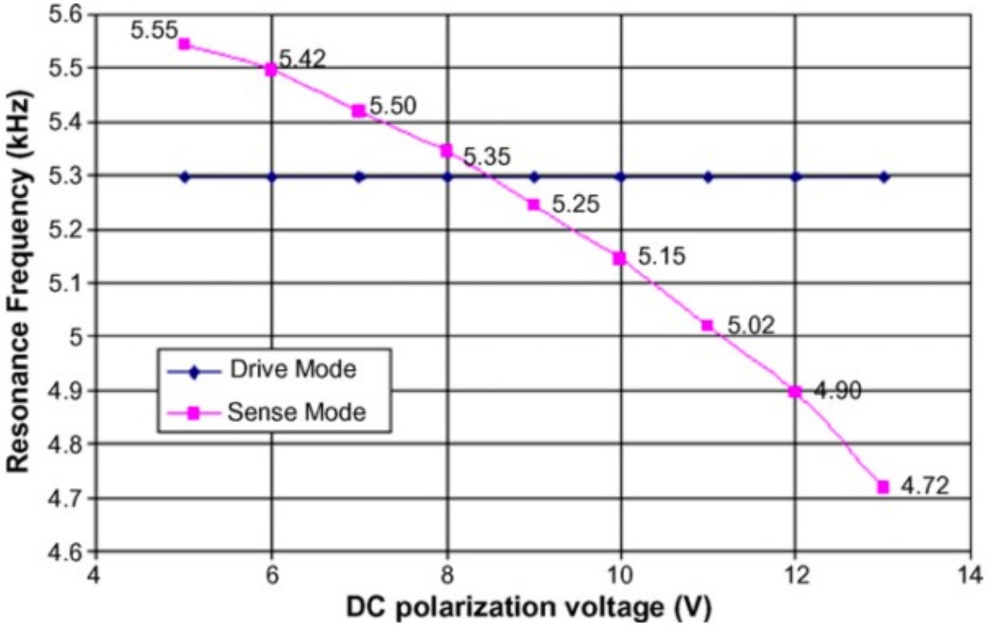


Fig. 8. Demonstration of effective frequency tuning for the sense-mode of the SOI gyroscope. The sense-mode resonance frequency of the gyroscope can be reduced from 5.55 kHz at 5 V dc down to 4.72 kHz at 13 V dc by negative electrostatic spring constant.

Comb drive (“drive mode”) = no spring softening

Parallel plates (”sense mode”) = significant spring softening

For actuation comb drive

Q = 460

k = 140 N/m

F_{res} = 5300 Hz

Capacitance: 1000 fF

Finger width: 2 μm

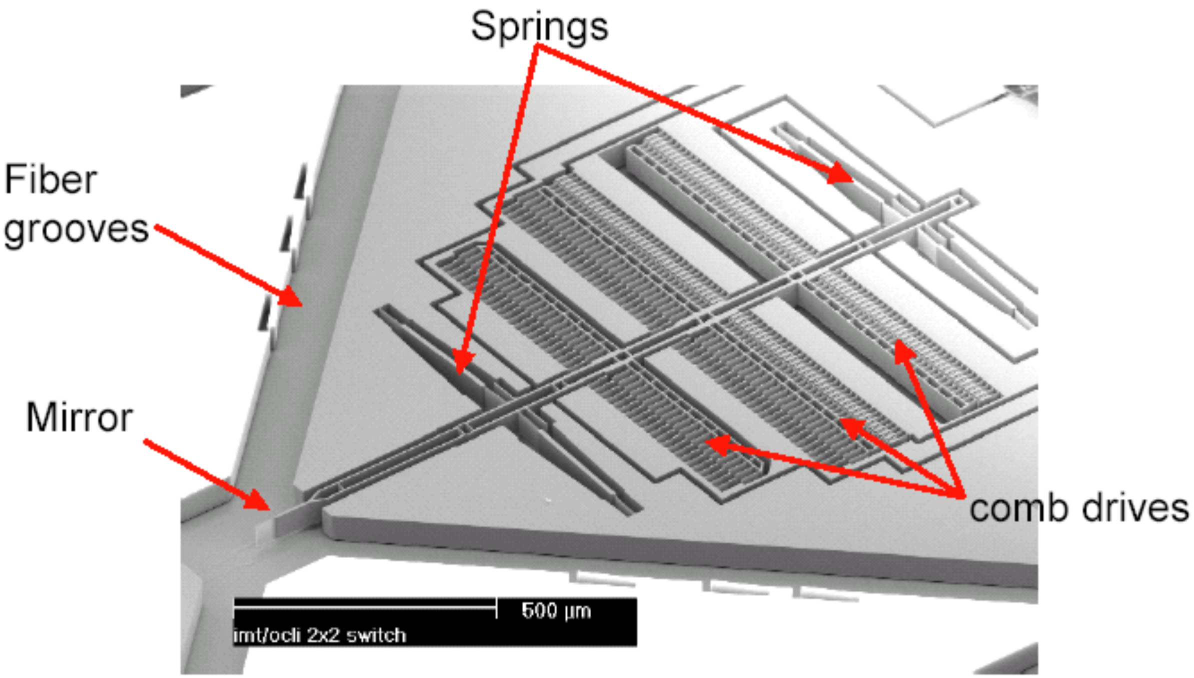
Gap: 2.6 μm

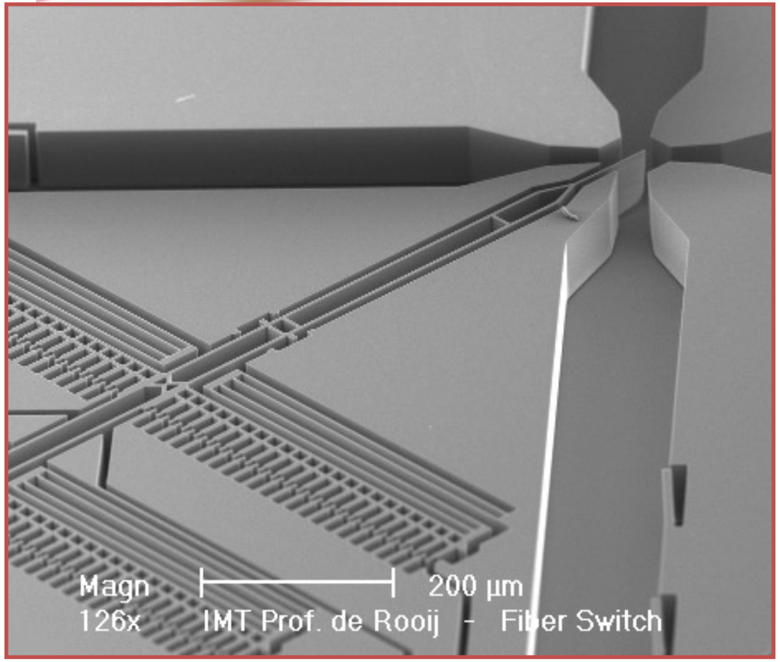
Thickness: 25 μm

Amplitude of Motion = 19 μm

Alper, et al. (2007). “A high-performance silicon-on-insulator MEMS gyroscope operating at atmospheric pressure”. *Sensors and Actuators, A: Physical*, *135*(1), 34. http://doi.org/10.1016/j.sna.2006.06.043

Bistable spring + comb drive for optical switch (Sercalo.com)





C. Marxer & de Rooij, IMT ([www.sercalo.com)](http://www.sercalo.com)

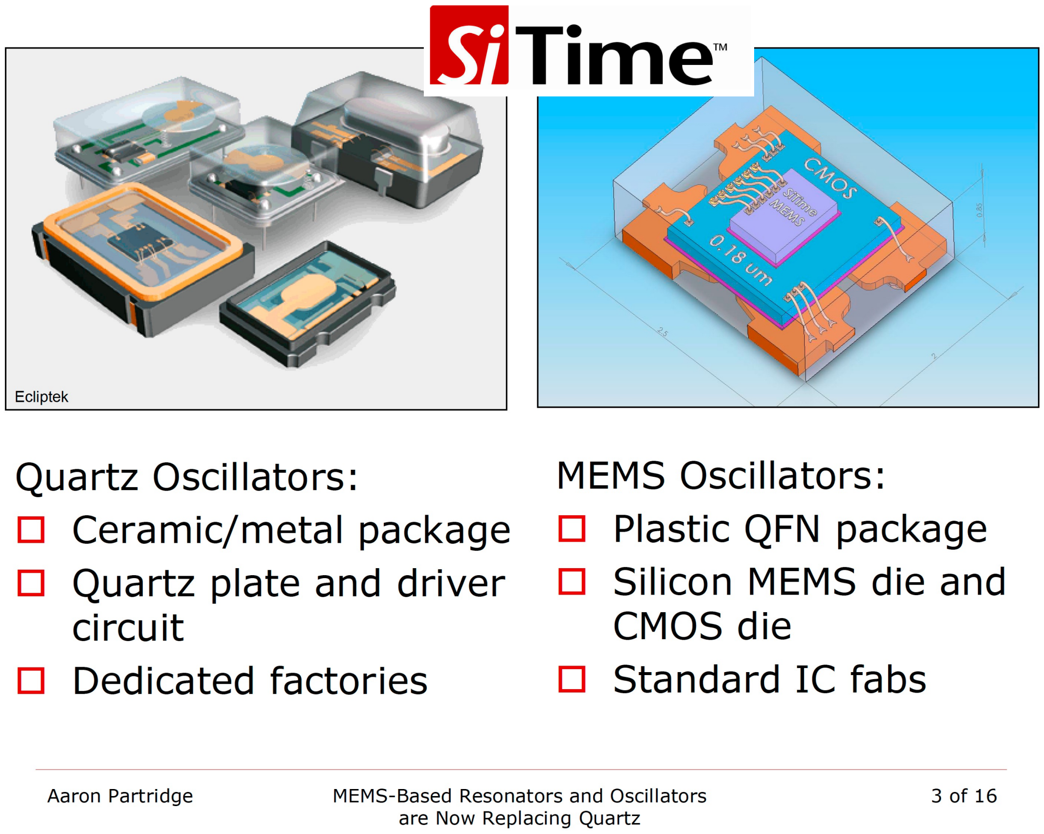
Marxer, C. R., Griss, P., & De Rooij, N. F. (1999). A Variable Optical Attenuator Based on Silicon Micromechanics. *IEEE Photonics Technology Letters*, *11*(2), 233–235. http://doi.org/10.1109/68.740714

Resonators:

coupling electrostatics + mechanics

MEMS Si resonators: excellent oscillators, poor filters

- Growing market: silicon resonator (as an oscillator) can replace Quartz:
  - much less bulky,
  - reduced part count,
  - possible integration with CMOS.
- Used ~~as filter~~ or frequency reference.
  - Energy from electrical to mechanical to electrical.
  - Using MEMS high-Q resonance
  - (For filters: use thin piezo materials like AlN)
- Challenges:
  - Linearity
  - Power handling (for filters)
  - **Frequency stability over temperature and time**
  - Packaging
  - Cost (need for HV bias)



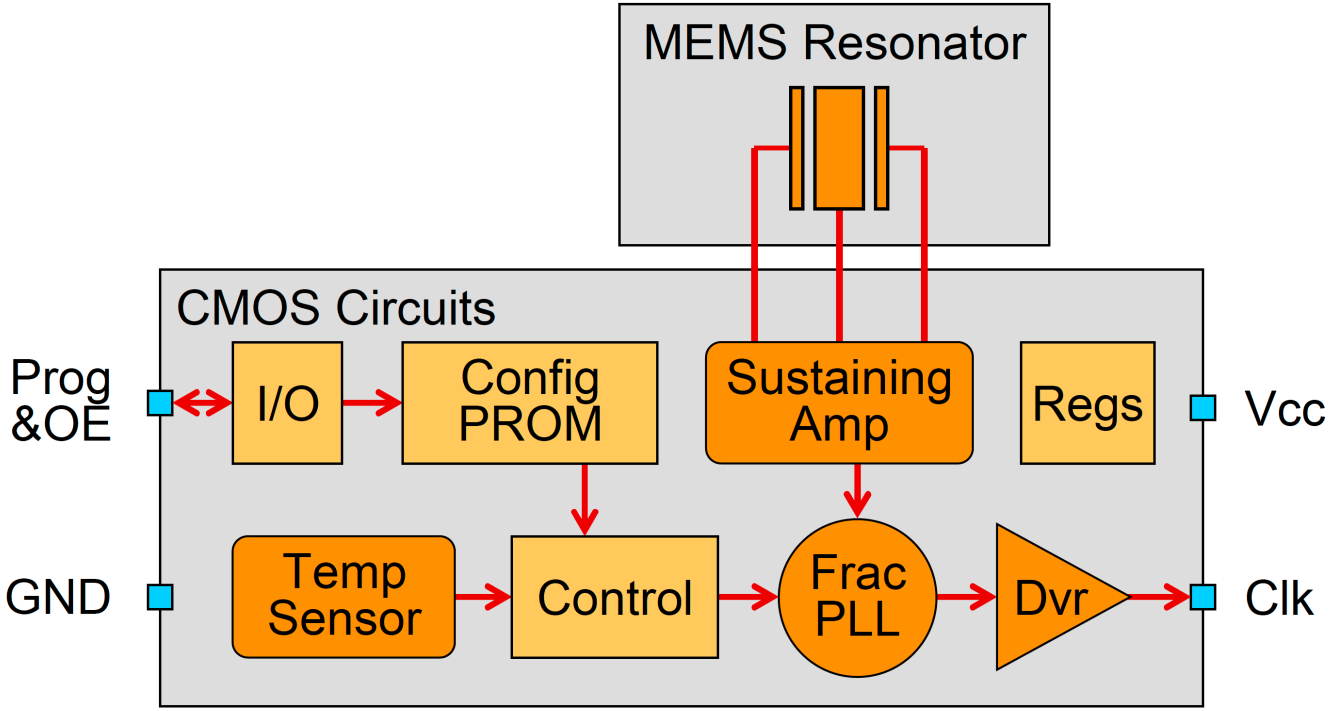
Quartz Oscillators:
- ☐ Ceramic/metal package
- ☐ Quartz plate and driver circuit
- ☐ Dedicated factories

MEMS Oscillators:
- ☐ Plastic QFN package
- ☐ Silicon MEMS die and CMOS die
- ☐ Standard IC fabs

Aaron Partridge MEMS-Based Resonators and Oscillators are Now Replacing Quartz 3 of 16

https://www.sitime.com/sites/default/files/gated/Aaron-Partridge-ISSCC-Tutorial-Slides.pdf

From Resonator to oscillator: need more than MEMS !



https://www.sitime.com/sites/default/files/gated/Aaron-Partridge-ISSCC-Tutorial-Slides.pdf

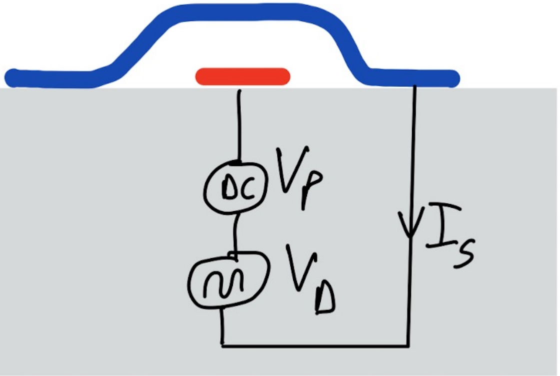
- It’s the CMOS that really makes it work! (but we’ll only discuss the MEMS here)
- SiTime sells complete oscillator: not just the Silicon resonator, which is useless on its own

Electro-mechanical resonator : ES Actuation

Applied voltage: $V = V_p + V_d \sin \omega t$ *V_p: polarization voltage* *V_d: drive amplitude (signal) V_p>>V_d*

Drive amplitude $F_{drive} = -\frac{1}{2}\left(V_p + V_d \sin\left(\omega_0 t\right)\right)^2 \frac{dC}{dx}$

Assume small displacement (linear), then $\frac{dC}{dx} = -\frac{\varepsilon_0 A}{d^2} \propto L^0$



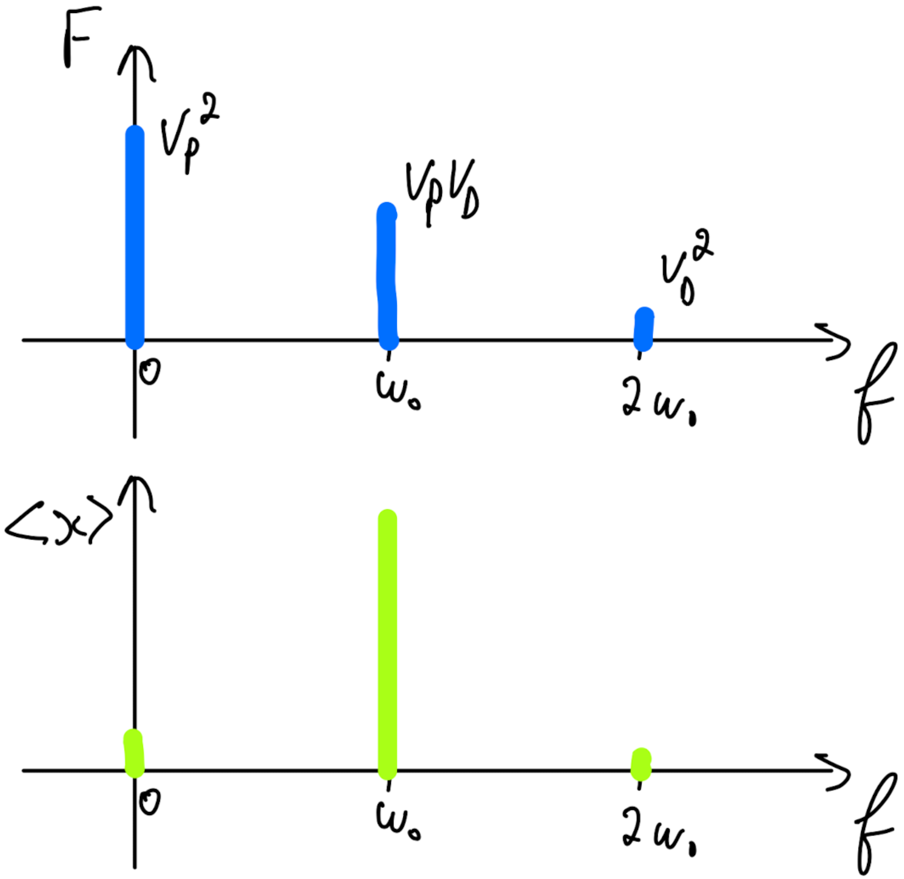
F_{drive} has components at: DC, ω_0 and $2\omega_0$

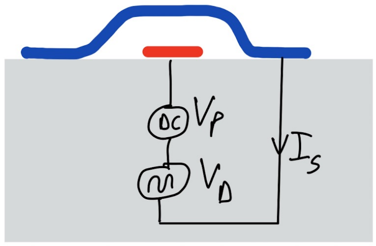
Force amplitude at ω_0 $\left|F\right|_{\omega 0} = -V_p V_d \frac{dC}{dx}$ $\propto L^0$ (if d scales as plate dimension)

$$\left|x_0\right| = Q\frac{F_{drive}}{k_{eff}} = \frac{QV_pV_d}{k_{eff}}\frac{dC}{dx}$$

Q is quality factor

(we care about what happens at resonance, e.g. as a frequency source)


F
V_p^2
V_pV_D
V_D^2
0
ω_0
$2\omega_0$
f
$\langle x \rangle$
0
ω_0
$2\omega_0$
f


DC
V_P
V_D
I_S

Electro-mechanical resonator: ES Detection

$$i_s = \frac{d(VC)}{dt} = V\frac{dC}{dt} + C\frac{dV}{dt}$$

$$= (V_p + V_D \sin\omega t)\underbrace{\frac{dC}{dt}}_{\hookrightarrow \frac{dC}{dx}\cdot\underbrace{\frac{dx}{dt}}_{\hookrightarrow \omega_0|x_0|}} + C\cdot\omega_0 V_d \cos\omega t$$

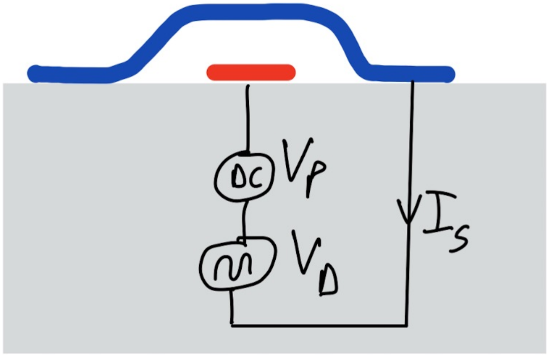
since $V_p \gg V_d$

$$i_s \approx V_p \frac{dC}{dt}$$

$$|i_s| = V_p \frac{dC}{dx}\omega_0|x_0|$$

$$|x_0| = Q\frac{F_{drive}}{k_{eff}} = \frac{QV_pV_d}{k_{eff}}\frac{dC}{dx}$$

$$\boxed{|i_s| = \frac{QV_p^2V_d}{k_{eff}}\omega_0\left(\frac{dC}{dx}\right)^2}$$



Input: voltage V_p and V_d (Polarization and ac)

generates mechanical motion,
filtered with device high Q

Output: current I_s

$$\eta = V_p\frac{dC}{dx}$$

Electro-mechanical resonator: ES Detection

Motional resistance R_x

$$\left|i_s\right| = \frac{QV_p^2V_d}{k_{eff}}\omega_0\left(\frac{dC}{dx}\right)^2$$

$$R_x = \frac{V_d}{i_s} = \frac{k_{eff}}{Q\omega_0 V_p^2\left(\frac{dC}{dx}\right)^2} = \frac{k_{eff}\cdot d^4}{Q\omega_0 V_p^2 \varepsilon_0^2 A^2}$$

$$R \propto L$$

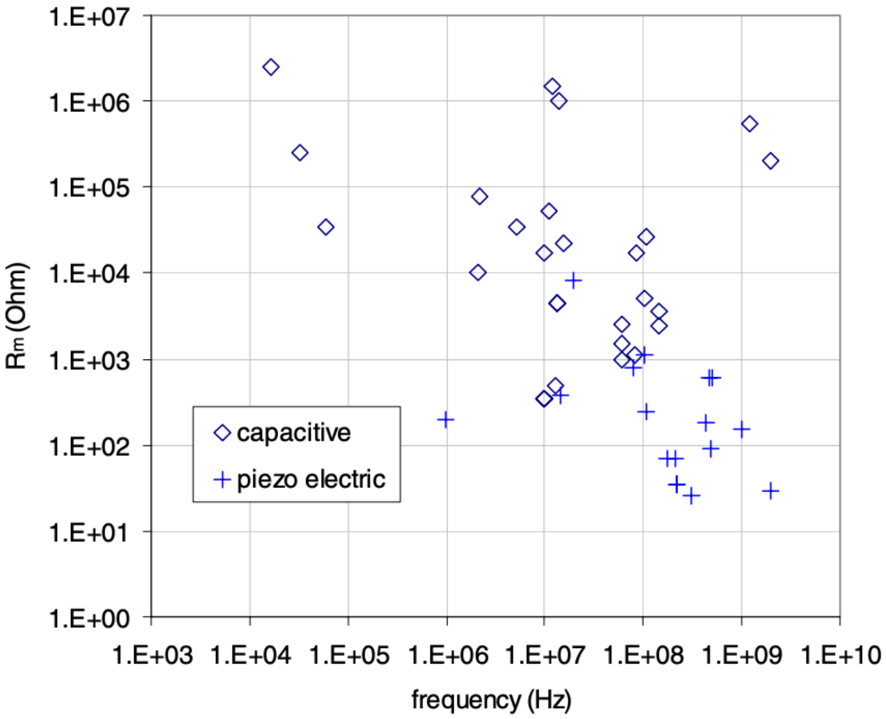
$$R \propto d^4$$

$$R_x \propto \frac{1}{Q}\cdot\frac{1}{V_p^{\,2}}$$

R_x is related to losses from anchor, friction, etc.
often of order MΩ for capacitive devices (or need very high V_P if want lower R_x)

R_x values of MΩ are tolerable for oscillator, because the oscillator is driven
But ideally R_x<1 Ω for a filter …

Motional resistance $R_x = \dfrac{k_{eff}\, d^4}{Q \omega_0 V_p^2 A^2}$



J T M van Beck and R Puers, “A review of MEMS oscillators for frequency reference and timing applications”, JMM 22 (2012) 013001

Table 4. Literature overview of resonance frequency, resonance mode, *Q*-factor, gap width, impedance, bias voltage and transducer gap width of capacitive resonators.

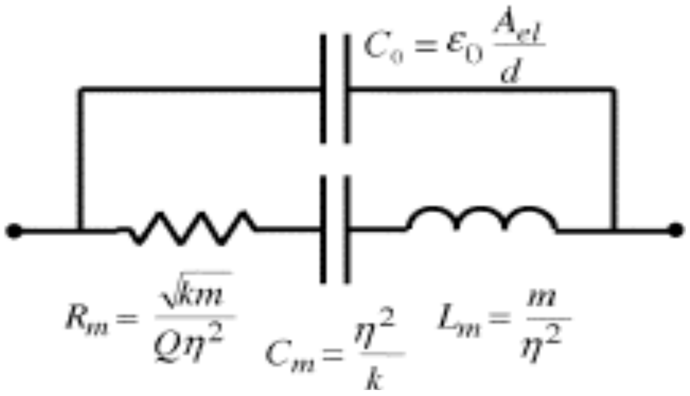
Reference	Center frequency (Hz)	*Q*-factor	Mode	R_m (Ω)	Bias voltage (V)	Resonator gap (nm)
[114]	1.65E+04	51 000	Si flex	2.50E+06	32	2000
[115]	3.20E+04	40 000	Si flex	2.50E+05	3	1000
[55]	6.00E+04	1000	Si flex	3.50E+04	28	1000
[116]	1.00E+06		Si flex		7	
[117]	2.06E+06	4050 000	Si bulk	1.00E+04	50	2000
[104]	2.18E+06	1160 000	Si bulk	7.64E+04	60	3000
[118]	5.10E+06	80 000	Si flex	3.50E+04	5	400
[119]	5.40E+06	2020 000	Si bulk			2000
[120]	6.30E+06	1600 000	Si bulk			2000
[121]	9.75E+06	3600	Si flex	1.75E+04	7	100
[62]	1.00E+07	1036	Si flex	3.40E+02	13	100
[62]	1.00E+07	1036	Si flex	3.40E+02	13	100
[122]	1.20E+07	180 000	Si bulk	1.50E+06	100	1000
[123]	1.30E+07	100 000	Si bulk	5.00E+02	20	180
[124]	1.31E+07	130 000	Si bulk	4.47E+03	100	750
[125]	1.31E+07	130 000	Si bulk	4.47E+03	100	750
[126]	1.40E+07	1500	Si flex	1.00E+06	130	1000
[66]	1.54E+07	4360	Si flex	2.20E+04	30	300
[47]	1.9E+07	220 000	Si torsion	1.2E+04	1	130
[127]	2.40E+07	53 000	Si bulk	2.10E+03	5	110
[67]	5.99E+07	6200	Si bulk	9.66E+02	16	32
[67]	6.10E+07	130 000	Si bulk	2.60E+03	16	92
[128]	6.12E+07	48 000	Si bulk	1.50E+03	12	80
[129]	8.12E+07	40 000	Si bulk	1.10E+03	10	65
[129]	8.59E+07	77 000	Si bulk	1.71E+04	7	170
[130]	1.03E+08	80 000	Si bulk	5.00E+03	18	200
[129]	1.07E+08	49 600	Si bulk	2.65E+04	10	170
[129]	1.44E+08	39 000	Si bulk	3.60E+03	50	135
[131]	1.45E+08	51 000	Si bulk	2.40E+03	14	77
[132]	1.20E+09	3700	Si bulk	5.60E+05	20	85
[79]	1.90E+09	1400	AlN bulk			
[133]	1.95E+09	8000	Si bulk	2.00E+05	20	
[86]	4.50E+09	11 000	Si bulk			

Equivalent circuit

Electromechanical transduction factor $$\eta = V_p \frac{dC}{dx}$$

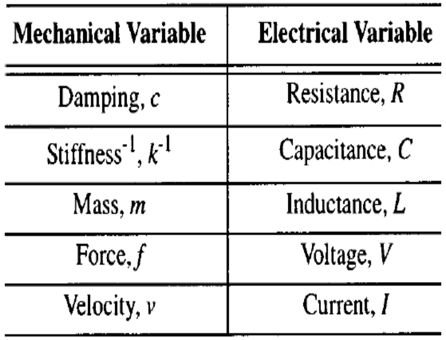
$$R_x = \frac{V_d}{i_s} = \frac{k_{eff}}{Q\sqrt{k_{eff}/m} \cdot \eta^2} = \frac{\sqrt{k_{eff} m}}{Q \cdot \eta^2}$$

$$L_x = \frac{m}{\eta^2} \qquad C_x = \frac{\eta^2}{k_{eff}}$$



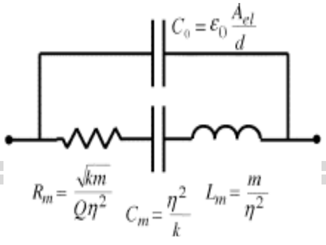
C_0=parasitic capacitance (ie a real capacitance)

TABLE I
MECHANICAL-TO-ELECTRICAL
CORRESPONDENCE IN THE CURRENT ANALOGY



Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Series and parallel resonances



Series resonance f:

$$f_s = \frac{1}{2\pi\sqrt{L_x C_x}} = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} \qquad Q = \frac{1}{2\pi f_s R_x C_x}$$

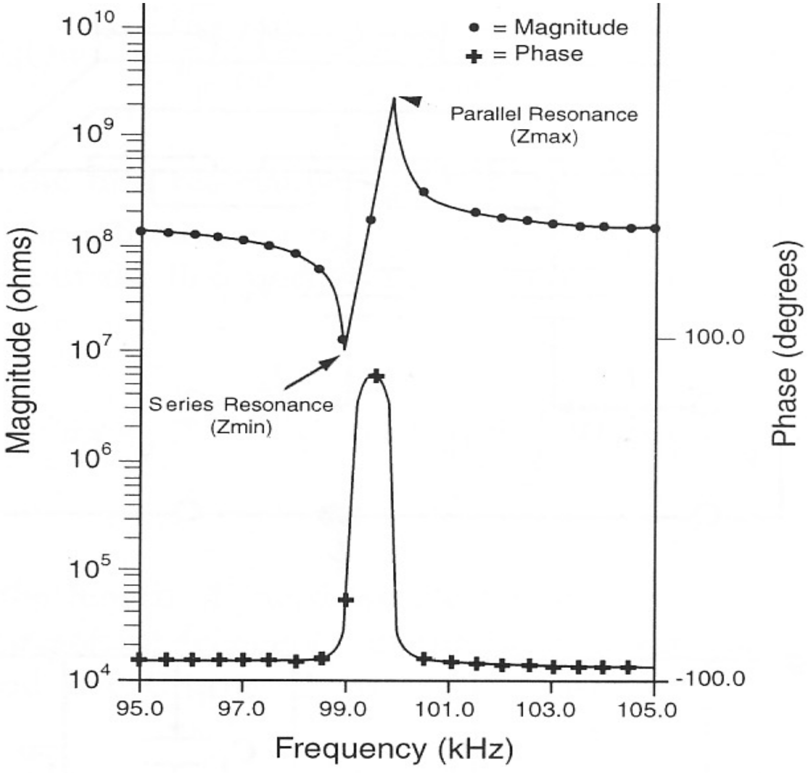
Parallel resonance f:

$$f_p = \frac{1}{2\pi}\frac{1}{\sqrt{L_x C_x\left(1 + C_x / C_0\right)}} = f_s\sqrt{1 + C_x / C_0}$$

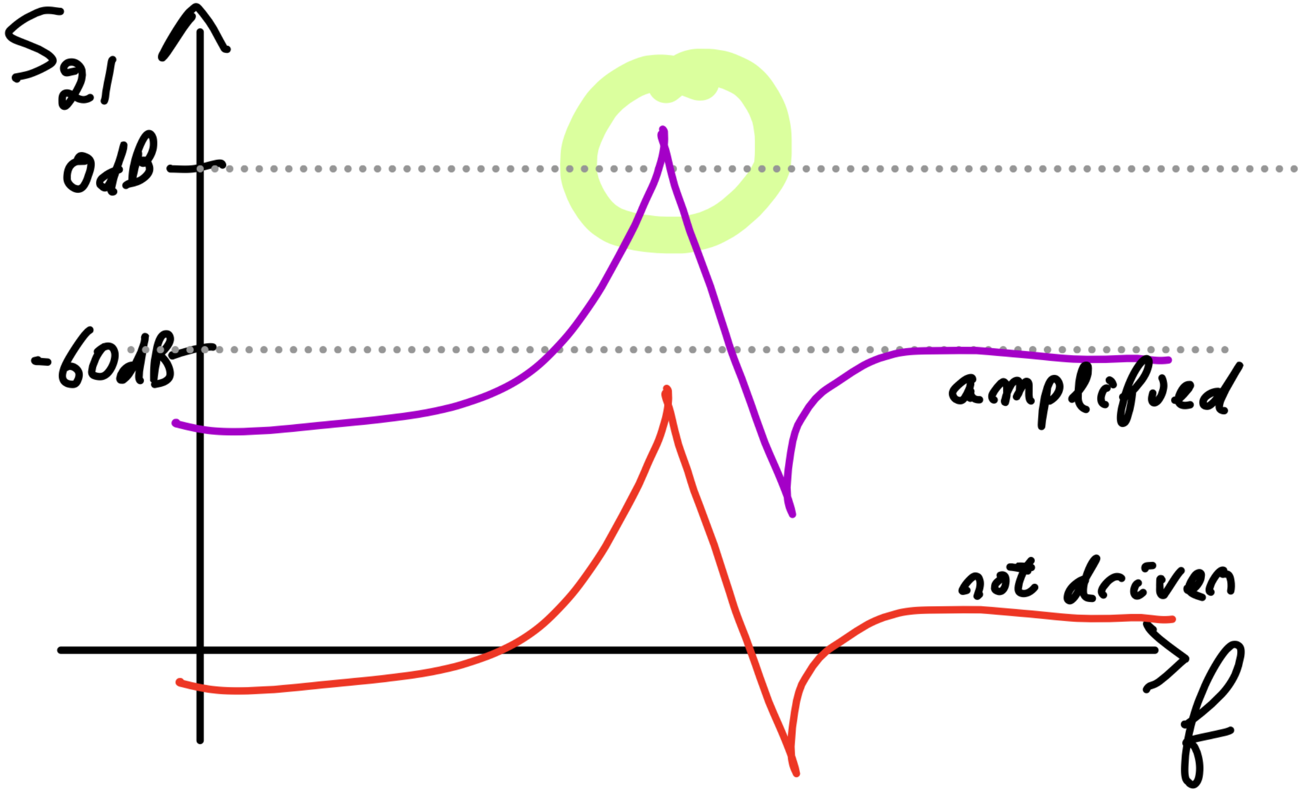
- At series resonance, MEMS **motion** and **current** are **maximal.** Impedance Z is given by R_x motional. f_s depends on polarization voltage:

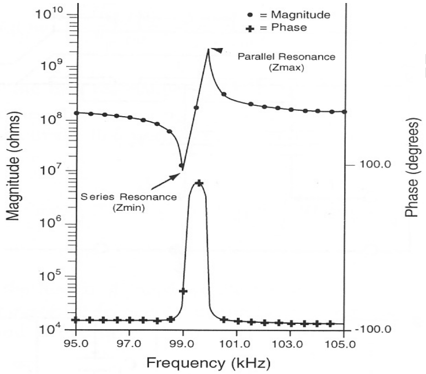
$$k_{eff} = k_{eff}\left(V_p\right)$$

- At parallel resonance, current due to motion and due to drive voltage almost cancel. **High resistive** impedance Z. Effectively current loop with external capacitance



We use the series resonance for oscillator





S_{21} is the **forward transmission** (from port 1 to port 2), i.e. proportional to 1/Z

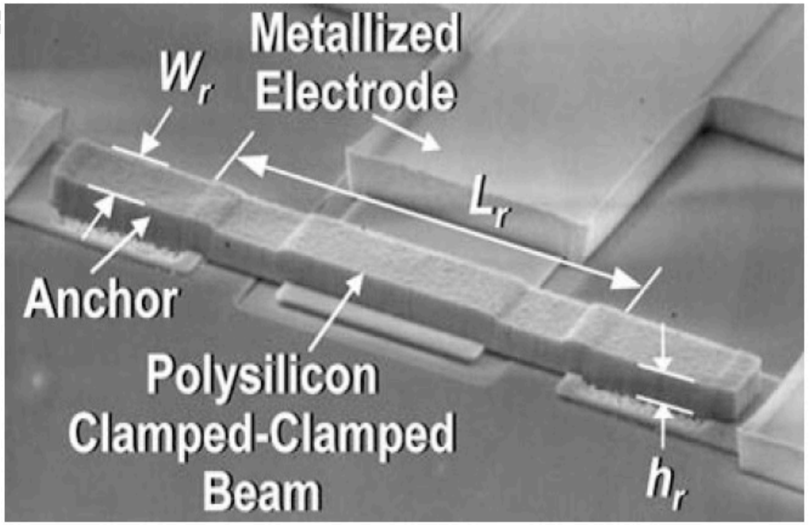
Bending mode resonators

- Early work used poly-Si clamped-clamped beams with gaps of 100 nm

- Cantilevers work in bending. Stiffness scales with t/L

- Cantilever: quickly reach non-linear motion
- Clamped-clamped : highly T dependent frequency

- Compare mechanical energy $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ with k_BT...
- Mechanical energy essentially scales with area: $\propto L^2$
  - For kHz beams, at resonance $E_{mech} \gg k_BT$
  - If scale down cantilever to increase f_{res} to tens of MHz beams, then $E_{mech} \sim k_BT$



C. Nguyen et al

Frequency scaling and thermo-mechanical noise was covered in mechanical scaling chapter.

Example

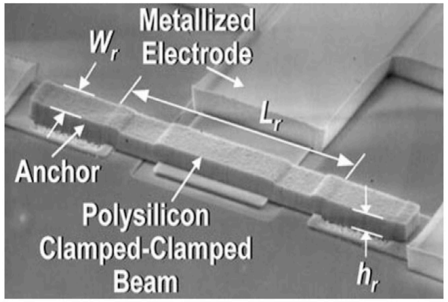
$k_BT = 4.10^{-21}$ J at room temperature

At resonance, using motion with Q factor

For cantilever at 34 kHz, $\frac{1}{2}kx^2 \approx 10^{-18}$ J

For bulk mode at 13 MHz, $\frac{1}{2}kx^2 \approx 10^{-11}$ J

Cantilever at MHz will be swamped by thermal noise…



k=10^{-4} N/m

x_{max} = 0.1 µm

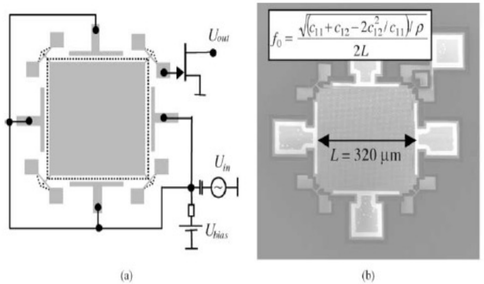


Fig. 1. Square-extensional microresonator (f_0 = 13.1 MHz and Q = 130 000). (a) Schematic of the resonator showing the vibration mode in the expanded shape and biasing and driving setup. (b) SEM image of the resonator.

k=2.10^{7} N/m

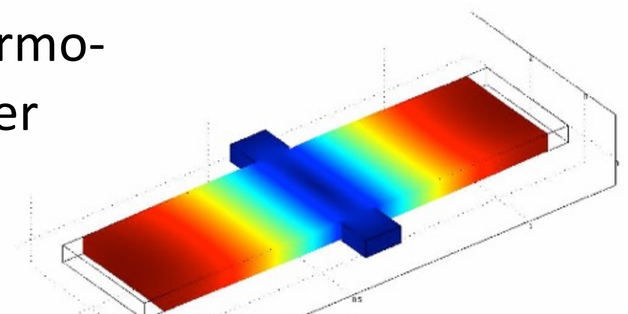
x_{max} = 0.5 nm

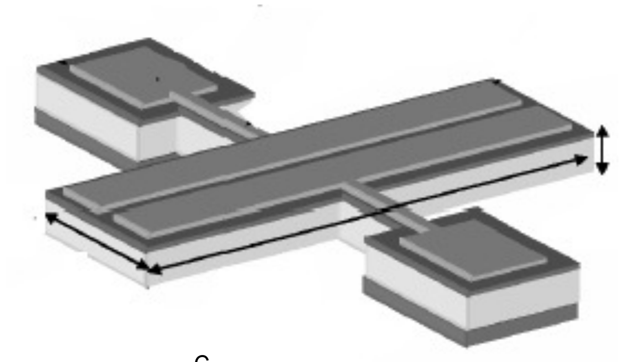
Bulk-mode resonators

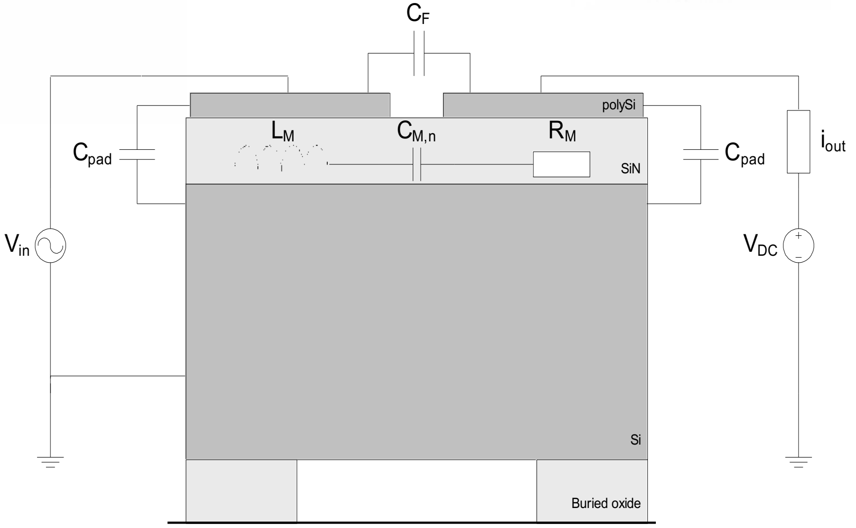
Allows Scaling up frequency without reducing area

- To avoid limitations related to thermo-mechanical noise, and to get higher frequencies, want to store a lot of mechanical energy
  - Large MEMS mass
  - High Q
  - High stiffness

- **So need bulk modes** (not bending modes)







S.A. Bhave, et al. “Silicon Nitride-On-silicon Bar Resonator Using Internal Electrostatic Transduction”, Transducers’05.

In-plane bulk-mode resonators (Lamé, breathing…)

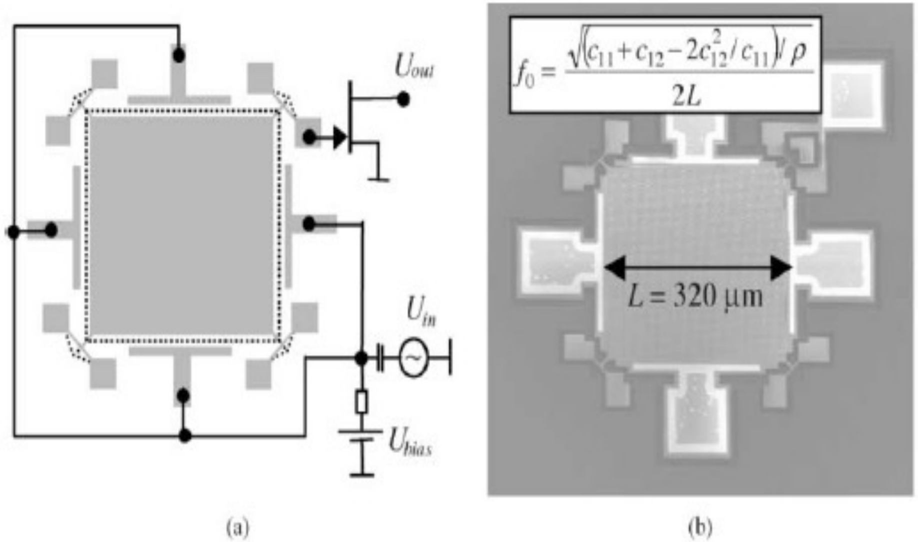


Fig. 1. Square-extensional microresonator ($f_0 = 13.1$ MHz and $Q = 130\,000$). (a) Schematic of the resonator showing the vibration mode in the expanded shape and biasing and driving setup. (b) SEM image of the resonator.

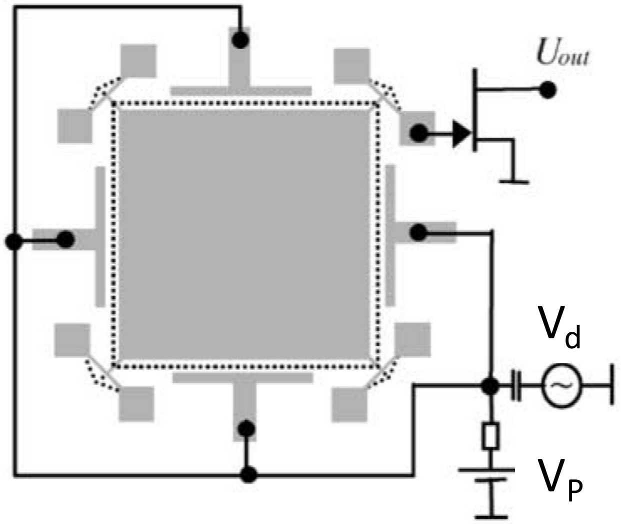
(Here 10 µm thick)

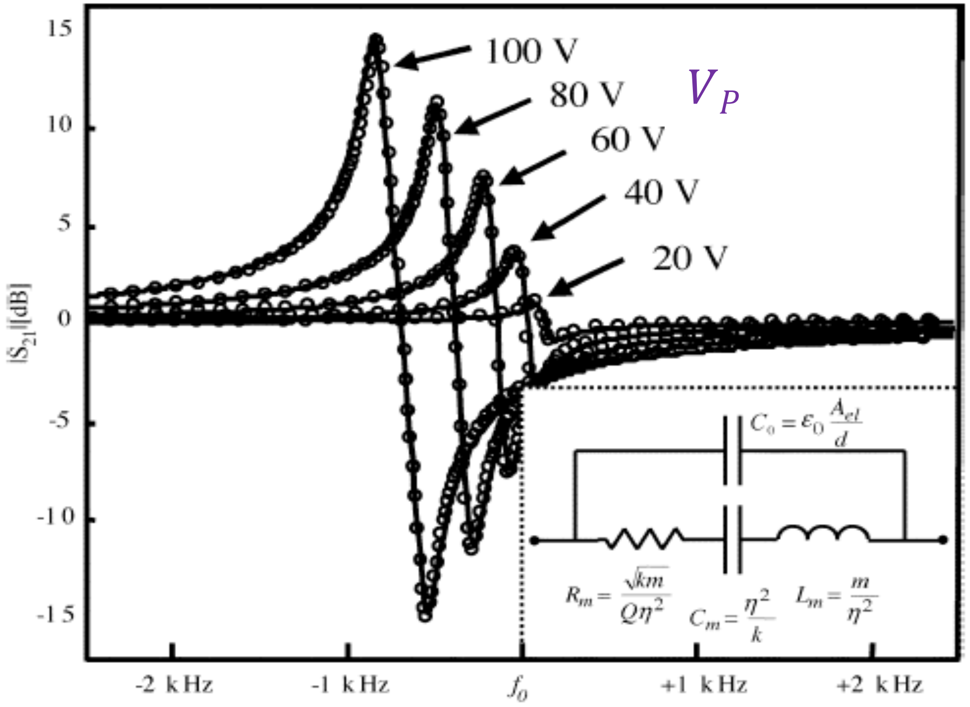
V. Kaajakari et al., « Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications », IEEE ELECTRON DEVICE LETTERS, VOL. 25, NO. 4, 2004

Bulk mode: large mass, high Q. One can make as thick as technology allows, without changing f_{res}

Spring softening: higher V_P means higher amplitude but lower frequency

$$\left|i_s\right| = \frac{QV_p^2V_d}{k_{eff}}\omega_0\left(\frac{dC}{dx}\right)^2$$





- Same “spring” softening issue we saw with parallel plate actuators: shift in frequency
- The mechanical springs are linear: the non-linearity is electrostatic, not mechanical

V. Kaajakari et al., “Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications”, IEEE Electron Device Letters, Vol. 25 (2004)

Here 50 mV AC on top of DC $V_{polarization}$

Figure of merit FOM=f.Q (for low phase noise)

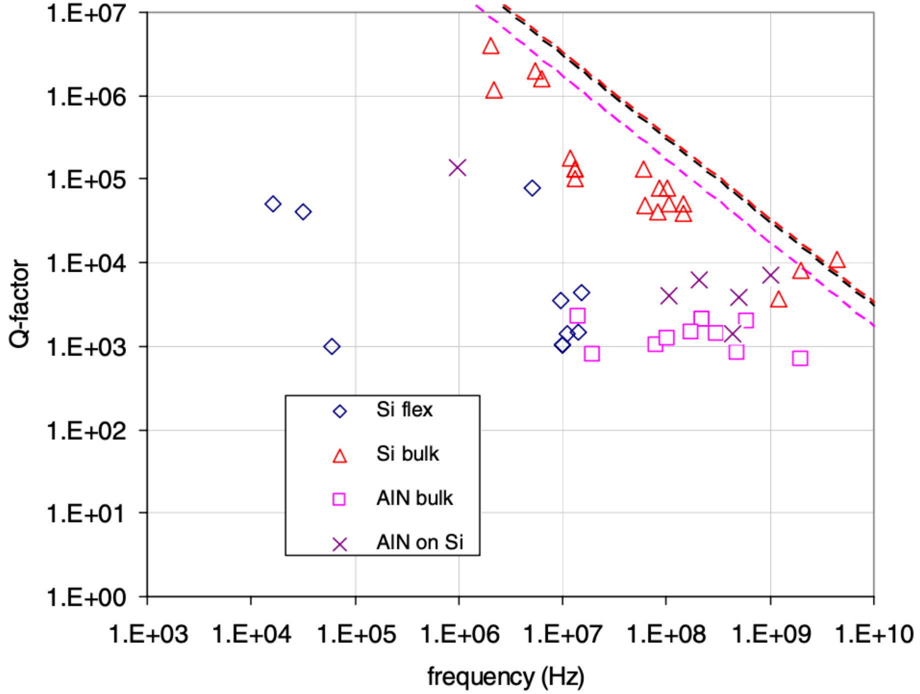


Figure 16. Overview of reported values of the unloaded *Q*-factor versus resonance frequency. The predicted maximum obtainable *f*–*Q* product is resembled by the dashed line for AlN (purple), quartz (black) and Si (red), respectively. Data are taken from table 5.

Single crystal Silicon:

- Good: High Q
- Bad: need high V_p

AlN

- Good: no DC voltage needed
- Bad: lower Q

J T M van Beck and R Puers, “A review of MEMS oscillators for frequency reference and timing applications”, JMM 22 (2012) 013001

Frequency Stability of a resonator: the key practical parameter

- Need stability of order ppm/°C
- Si has huge change in E_{Young} with T …

- T-dependence of material properties
  - Thermal expansion
  - Young’s modulus change (10x more important than thermal expansion)
    - Oxide gets stiffer with increasing T
    - Other materials gets softer with increasing T

- Packaging:
  - Moisture
  - Q (vacuum)

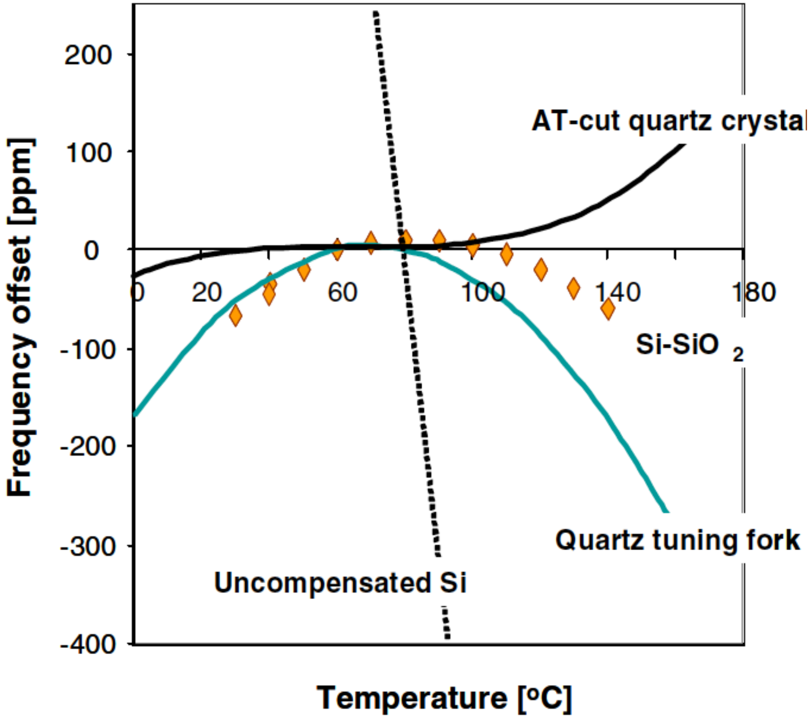


Figure 24. Frequency shift as a function of frequency for AT-cut quartz, tuning fork quartz, Si and oxidized silicon resonator.

J T M van Beck and R Puers, “A review of MEMS oscillators for frequency reference and timing applications”, JMM 22 (2012) 013001

5 ways to improve stability (T-drift)

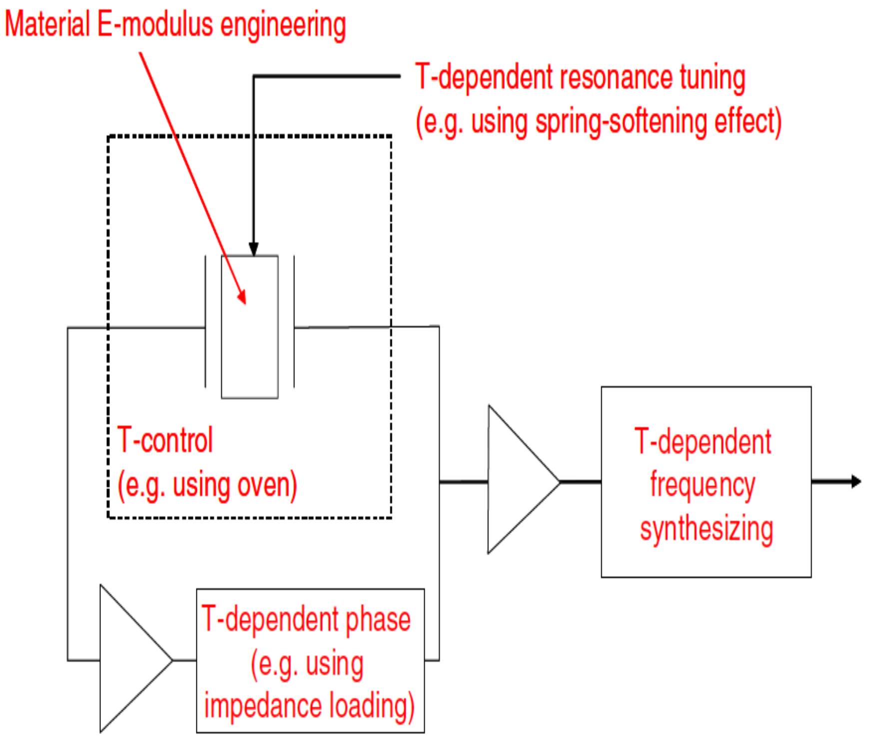


Figure 23. Five approaches exist for the compensation of the temperature-induced frequency drift of a MEMS resonator ranging from resonator level solutions to oscillator level concepts.

J T M van Beck and R Puers, "A review of MEMS oscillators for frequency reference and timing applications", JMM 22 (2012) 013001

$E_{modulus}$ engineering

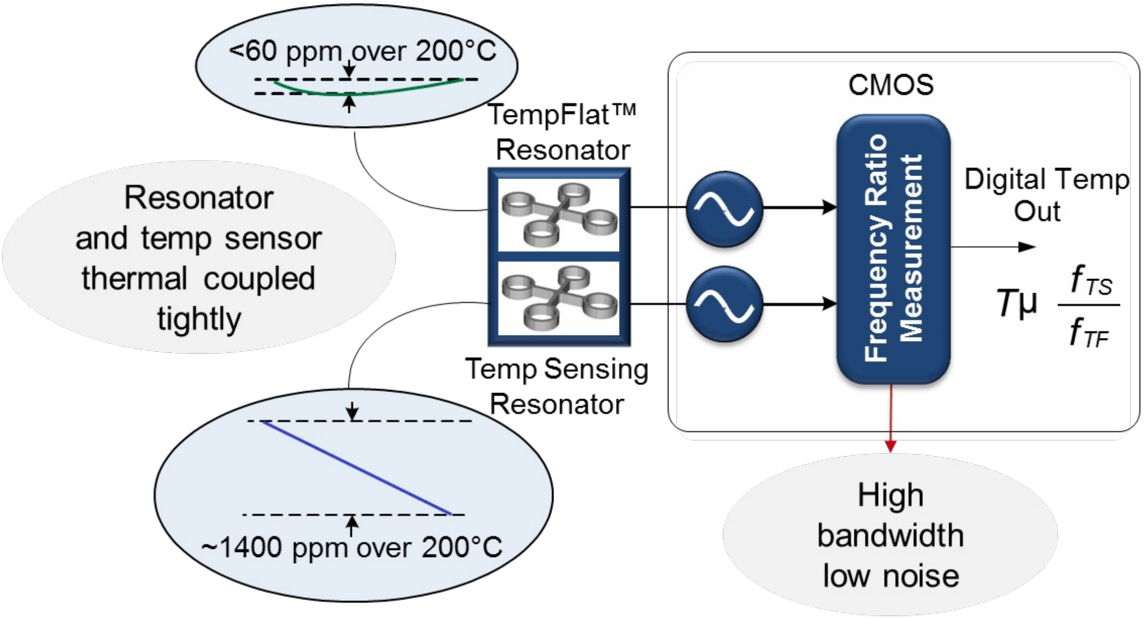
- **Ion implant Si (reason for SiTime success)**
- Grow SiO layer (but reduces Q factor, so not used in commercial solutions

Power consumption for mobile devices! So prefer passive (eg E-modulus engineering) to oven

SiTime numbers for Stability (ion-implanted Si)

SiTime “TempFlat” MEMS alone gives stability of silicon MEMS oscillator of better than ±60 ppm over temperature from -40◦C to +85◦C. i.e. 100x better than the Si material

With on-chip T sensors, SiTime claims ±1 ppm between -40◦C and +85◦C.



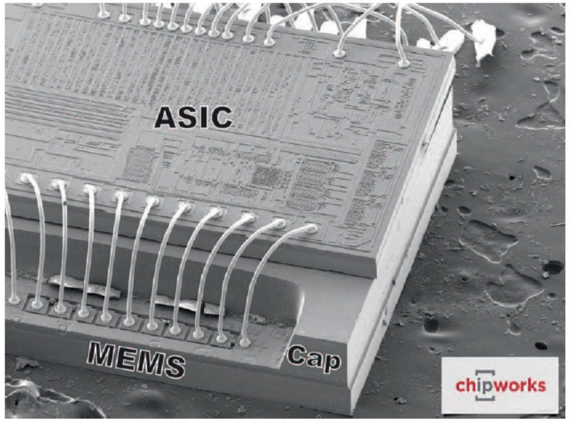
https://www.sitime.com/sites/default/files/gated/TechPaper-DualMEMS-Temp-Sensing-2018.pdf

CMOS integration: MEMS first, or MEMS last? Or hybrid

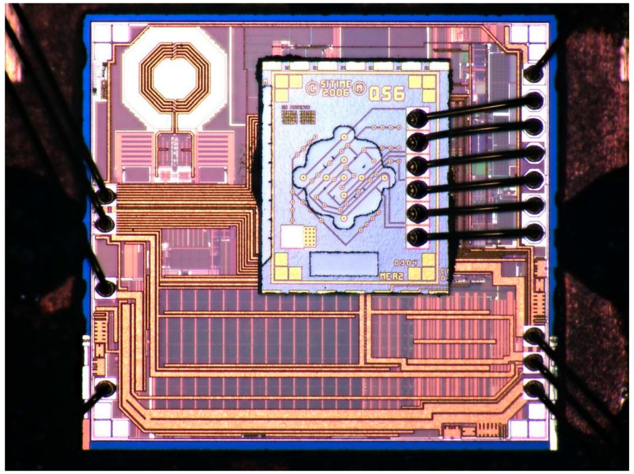
A. MEMS first, then CMOS (eg early Analog devices accelerometers)

B. CMOS, then MEMS (eg TI DMD)

C. **Separate MEMS and CMOS processes, then bond** (most current devices)



STMicroelectronics LIS331DLH 3-axis accelerometer



SiTime

Fischer, A. C. *et al.* Integrating MEMS and ICs.
Microsystems & Nanoengineering **1**, 15005 (2015).

Resonators

- Piezos (eg in FBAR) are better for high frequency filters (mechanics + electrical + piezo), eg AlN
    - Lower $R_{motional}$
    - Higher coupling
    - High bandwith
    - No $V_{polarization}$. Simpler circuit

- Piezos can offer higher electromechanical efficiency for resonators